

Kingsmead School Numeracy Strategy 2023/24

The purposes of our whole-school numeracy policy is:

- to develop, maintain and improve standards in numeracy across the school;
- to ensure consistency of practice including methods, vocabulary, notation, etc.;
- to indicate areas for collaboration between subjects;
- to assist the transfer of students' knowledge, skills and understanding between subjects.

What is numeracy

In its simplest form, numeracy (or being numerate) is "... **the ability to confidently use maths in daily life and the workplace.** Numeracy does not include more complex skills, such as algebra, but it does mean being confident enough to choose and apply basic maths in real-life situations, and these can often be complex. Numeracy is as important as literacy - it's sometimes called 'mathematical literacy' - and we need both to get on in life."

(National Numeracy)

Kingsmead School is committed to developing the competence and confidence of all its students with basic maths and ensuring they leave being functionally numerate.

A Kingsmead student will be considered numerate if they:

- Have a sense of the size of number.
- Know by heart basic number facts such as number bonds, times-tables, doubles and halves, and recall them rapidly.
- Use what they know by heart to figure out an answer mentally.
- Calculate accurately, both mentally and with pencil and paper, and be able to draw on a range of strategies.
- Use a calculator appropriately.
- Make sense of number problems and recognise the operations needed to solve them.
- Know for themselves that their answers are reasonable and have strategies for checking.
- Explain their methods and reasoning, using correct terminology.
- Suggest suitable units for measurement and make sensible estimates of measurements.
- Explain and make sensible predictions from numerical data in a graph, chart or table.

Developing, maintaining and improving standards in numeracy

The numeracy standards of students will be monitored using low-stake termly tests (aligned with the school's assessment cycles). The skills will be addressed in weekly starters.

These tests will be differentiated by year group and will assess the students' knowledge and skills in the following areas:

- Rounding
- Ordering numbers
- Converting decimals to fractions
- Finding the median
- Special types of numbers
- Negative numbers

In addition to the tests, we are aware of there being issues surrounding units of measure, displaying data, rearranging formulae and timestables. These areas will be addressed by use of Sparx monitoring as well as whole school numeracy visual aids and work with the Science and Geography departments.

Analysis of these tests (as well as GL progress tests and the Maths Faculty's own internal assessments) will be used to identify those students who may require extra input and support. These students will then receive targeted intervention during mathematics lessons and tutor time.

Raising Standards in numeracy across our school will not be solely judged in increased test percentages. We will evaluate the students' ability to **transfer mathematical skills into the lessons of other subject areas and tutor time**, applying techniques to problem solving. Their confidence in attempting this is initially as important as achieving the correct solution. Student voice, work sampling and lesson observation will be the main processes for evaluating the success of our practice. *The transferrable skills already identified in the table below.*

		D&T	Biology	Chemistry	Physics	Business	Computer Science	Geography	PE	History	Art	English	MFL
2.1	Mental methods	Y	Y	Y	Y	Y	Y	Y	Y	Y			
2.2	Written methods	Y	Y	Y	Y	Y	Y	Y	Y	Y			
2.3	Number properties		Y	Y	Y								
2.4	Standard form		Y	Y	Y								
2.5	Other number bases					Y							
2.6	Estimation and accuracy		Y	Y	Y			Y	Y				
2.7	Fractions		Y	Y	Y			Y	Y				
2.8	Percentages		Y	Y	Y	Y		Y	Y	Y			
2.9	Ratio and proportion	Y	Y	Y	Y			Y			Y		Y
2.10	Directed numbers		Y	Y	Y			Y					
2.11	Coordinates							Y					
2.12	Inequalities		Y	Y	Y								
2.13	Shapes	Y							Y		Y		
2.14	Area, perimeter and volume	Y			Y								
2.15	Units of measure	Y	Y	Y	Y		Y	Y	Y	Y		Y	
2.16	Compass directions and bearings							Y	Y				
2.17	Algebra	Y	Y	Y	Y	Y	Y	Y	Y				
3.1	Collecting and recording data		Y	Y	Y			Y	Y				
3.2	Displaying data	Y	Y	Y	Y	Y		Y	Y	Y		Y	

Numeracy across the school

Numeracy is not the sole responsibility of the Maths Faculty. All subjects can contribute to the development and enhancement of students' numeracy skills including their ability to describe and explain their strategies and reasoning in real-life situations.

Fundamental to the development and enhancement of students' numeracy skills, will be **consistent practice** across the school.

Work has already begun by looking at SoLs across the school and liaising with relevant subject leads. Continued efforts will allow us to:

- confirm where numeracy skills are already being used in other subjects
- identify new opportunities in other subjects for:
 - a. reinforcing and extending mathematical skills;
 - b. developing mathematical vocabulary and the use of correct terminology and notation;
 - c. teaching a range of computation methods and ways of recording them – *including mental, written and calculator methods*, asking students to explain their methods and discussing with them which methods are best suited for particular purposes.
- provide assistance and advice, so that correct and common strategies (methods etc.) are used in all subjects.
- provide information to other subject teachers on appropriate expectations of students and difficulties likely to be experienced in various age and ability groups.
- align SoW and ensure that students have appropriate numeracy skills by the time they are needed for work in other subject areas.
- seek opportunities to use topics and examination questions from other subjects in Maths lessons.

Areas of collaboration between Maths and other subjects will include:

- Mental Arithmetic Techniques
- Methods of calculation and rearranging formulae
- Use of calculators
- Mathematical vocabulary
- Use of metric measures
- Use of charts and graphs

Members of the Maths Faculty will also support colleagues in other subjects to develop and enhance the students' numeracy skills by running numeracy CPD sessions and bespoke training for specific subjects where appropriate.

Students in all lessons should:

- Make correct use of mathematical vocabulary and notation when providing oral and written answers or asking questions.
- Present ideas and information they have collected in the form of charts and tables.
- Interpret, describe and explain their work and not simply reproduce graphs, tables and charts or statements concerning percentages and other numerical data.
- Set their work out systematically and with care. Where there are calculations, these should always be set out so the method used is clear. Where there are graphs these should always be correctly labelled and have a title.

Teachers of all subjects should:

- Have regard for the whole school numeracy policy in their planning of lessons.
- Use and explain mathematical vocabulary whenever it will enhance students' knowledge, skills and understanding of the topic. The use of such vocabulary by students should be strongly encouraged.
- Emphasise the use of mental calculation when it is sensible to do so.
- Encourage students to estimate when appropriate.
- Use and explain appropriate calculations and data whenever it will enhance students' knowledge and understanding of the topic. The explanation should be in line with whole school policy.
- Choose and use appropriate units of measure correctly.
- Demonstrate how to select a type of graph appropriate to the data provided.
- Correctly label a graph or chart.
- Correctly interpret a graph or chart and make appropriate comparisons where more than one graph illustrates the data.
- Explain the steps taken in the solution of a problem.
- Discuss and clarify why a particular method of calculation works.
- Recognise situations and problems in which numeracy skills and understanding can be used
- Assess the numeracy skills required to address these situations or problems – selecting the right tools and knowledge is a core part of being functionally numerate

SoLs should:

- Identify topics/areas requiring numeracy skills.
- Provide guidance to staff concerning approaches to the use of numeracy skills in their subject.
- Ensure that units of measure are written according to the correct convention.

2 Mathematical methods

With the exception of Mathematics, most subjects allow students to use calculators in GCSE examinations. This policy explores both non-calculator and calculator methods.

Throughout the policy, specific applications of numeracy in other curriculum areas are highlighted in yellow.

2.1 Mental methods

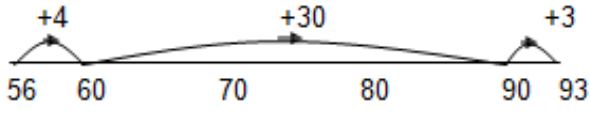
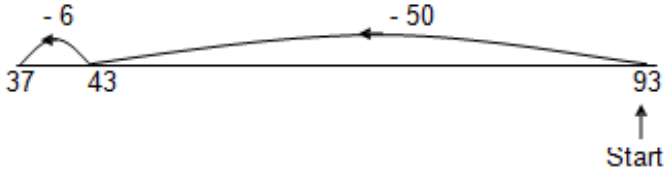
2.1.1 Addition – mental methods

$$54 + 27$$

Method 1	Method 2	Method 3
Add the tens, then the units, then add together	Split the number to be added into tens and units	Round up to the next 10, then subtract.
$50 + 20 = 70$	$54 + 20 = 74$	$54 + 30 = 84$
$4 + 7 = 11$	$74 + 7 = 81$	30 is 3 too many
$70 + 11 = 81$		$84 - 3 = 81$

2.1.2 Subtraction – mental methods

$$93 - 56$$

Method 1	Method 2
Count on	Break up the number being subtracted
Count on from 56 until you reach 93	e.g. subtract 50 then subtract 6
	
$4 + 30 + 3 = 37$	$93 - 50 = 43$
	$43 - 6 = 37$

Example - Chemistry

Mass is never lost or gained in chemical reactions. Mass is always conserved. The total mass of products at the end of the reaction is equal to the total mass of the reactants at the beginning.

The equation for a reaction is: $2 \text{CuCO}_3 + \text{C} \rightarrow 2 \text{Cu} + 3 \text{CO}_2$

A company calculated that 247 tonnes of copper carbonate (CuCO_3) are needed to produce 127 tonnes of copper (Cu) and 132 tonnes of carbon dioxide (CO_2) are released.

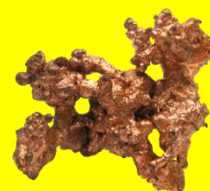
Calculate the mass of carbon (C) needed to make 127 tonnes of copper.

Total mass of products = mass of Cu + mass of CO_2 = $127 + 132 = 259$ tonnes

Total mass of products = total mass of reactants

$259 = \text{mass of } \text{CuCO}_3 + \text{mass of Carbon} = 247 + \text{mass of Carbon}$

Mass of Carbon = $259 - 247 = \mathbf{12 \text{ tonnes}}$



2.1.3 Multiplication – mental methods

It is essential that pupils know all of the times tables from 1x1 up to 10x10

If students are not fluent in their timetables, no matter what their age, they should practice them until they are. There are various websites and apps that will allow them to do this.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

39 x 6

Method 1	Method 2
Multiply by the tens then by the units	Round one of the numbers to make the calculation simpler, then use subtraction to correct your answer
$30 \times 6 = 180$	$40 \times 6 = 240$
$9 \times 6 = 54$	$40 = 39 + 1$
$180 + 54 = 234$	$240 - (1 \times 6) = 234$

2.2 Written Methods

2.2.1 Column addition and subtraction

Addition	Subtraction																																								
534 + 2678	7686 – 749																																								
Line up the digits in the correct “place value.” Begin by adding the units. Show working out.	Line up the digits in the correct place value. Begin by subtracting the units. 6 is smaller than 9, so take one “ten” from the eight “tens” to make the 6 units 16 units. Now continue subtracting. You will have to take 1 “thousand” from the “thousands” column when you subtract the “hundreds.”																																								
<table border="1"> <thead> <tr> <th></th> <th>Th</th> <th>H</th> <th>T</th> <th>U</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>5</td> <td>3</td> <td>4</td> </tr> <tr> <td>+</td> <td>2</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td></td> <td><u>1</u> 3</td> <td><u>1</u> 2</td> <td><u>1</u> 1</td> <td>2</td> </tr> </tbody> </table>		Th	H	T	U			5	3	4	+	2	6	7	8		<u>1</u> 3	<u>1</u> 2	<u>1</u> 1	2	<table border="1"> <thead> <tr> <th></th> <th>Th</th> <th>H</th> <th>T</th> <th>U</th> </tr> </thead> <tbody> <tr> <td></td> <td>6</td> <td><u>1</u> 6</td> <td>7</td> <td><u>1</u> 6</td> </tr> <tr> <td>-</td> <td></td> <td>7</td> <td>4</td> <td>9</td> </tr> <tr> <td></td> <td>6</td> <td>9</td> <td>3</td> <td>7</td> </tr> </tbody> </table>		Th	H	T	U		6	<u>1</u> 6	7	<u>1</u> 6	-		7	4	9		6	9	3	7
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2.2.2 Addition and subtraction of decimals

Addition of decimals					Subtraction of decimals							
53.4 + 26.78					78.9 – 7.49							
Line up the digits in the correct “place value.” Make sure the decimal points are lined up vertically. Begin by adding in the furthest column to the right. Show working out.					Line up the digits in the correct place value. Make sure the decimal points are lined up vertically. Fill in any gaps with “0”s. Begin subtracting in the furthest column to the right.							
	T		U	Tenths	Hundredths		T		U	Tenths	Hundredths	
	5		3	● 4			7		8	● 8	● 9	
+	2		6	● 7	8	-			7	● 4	9	
	1	8	1	0	● 1	8		7		1	● 4	1

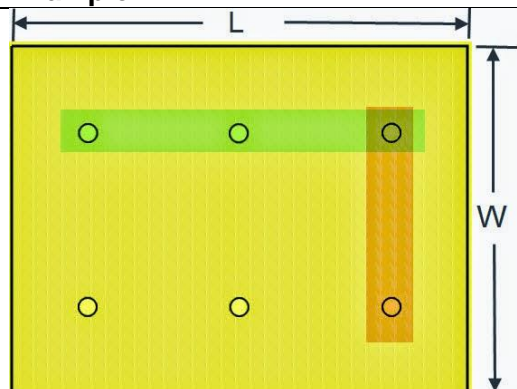
2.2.3 Multiplication

Method 1	Method 2	Method 3																																													
The column method	The grid method	Napier’s bones																																													
<p>This is the method that all students are now taught formally at Key Stage 2.</p> <p>56 x 34</p> <p>Line up the digits in their correct place values.</p> <p>Multiply the top number by the units in the bottom number, then by the tens in the bottom number* and so on.</p> <p>Add the products to get the final answer.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td><td></td><td>T</td><td>U</td></tr> <tr> <td></td><td></td><td>5</td><td>6</td></tr> <tr> <td></td><td>x</td><td>3</td><td>4</td></tr> <tr> <td></td><td>2</td><td>2</td><td>4</td></tr> <tr> <td>+</td><td>1</td><td>6</td><td>8</td></tr> <tr> <td></td><td>1</td><td>9</td><td>0</td></tr> <tr> <td></td><td></td><td>1</td><td>4</td></tr> </table> <p>*Note the highlighted “0” – this is because the highlighted “3” represents</p>			T	U			5	6		x	3	4		2	2	4	+	1	6	8		1	9	0			1	4	<p>56 x 34</p> <p>Separate each number into parts based on each digit’s place value.</p> <p>Write one number vertically and one number horizontally.</p> <p>Multiply the columns by the rows.</p> <p>Add the results.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>50</td><td>6</td></tr> <tr> <td>30</td><td>1500</td><td>180</td></tr> <tr> <td>4</td><td>200</td><td>24</td></tr> </table> <p>1500 + 180 + 200 + 24 = 1904</p>	x	50	6	30	1500	180	4	200	24	<p>847 x 6</p> <p>Write one number horizontally and the other vertically.</p> <p>Draw a grid with diagonals going from the top right to the bottom left corners of each box.</p> <p>Multiply the digits together then add along the diagonals.</p> <div style="text-align: center;"> <table border="1" style="margin: 0 auto;"> <tr> <td style="background-color: red;">8</td><td style="background-color: red;">4</td><td style="background-color: red;">7</td><td style="background-color: purple;">X</td></tr> <tr> <td style="background-color: orange;">4</td><td style="background-color: orange;">2</td><td style="background-color: orange;">4</td><td style="background-color: red;">6</td></tr> </table> <p style="margin-top: 10px;"> 5 0 8 2 </p> <p style="margin-top: 10px;"> ↑ ↑ ↑ </p> <p style="margin-top: 10px;"> 4+1=5 8+2=10 4+4=8 </p> <p style="margin-top: 10px;"> Write 0 Carry 1 </p> </div>	8	4	7	X	4	2	4	6
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<p>Calculator example – bacterial growth</p> <p>In the right conditions, a bacterium in the human body can split in two every 20 minutes. If a patient was infected with one E-coli bacterium, how many would there be after 24 hours?</p> <p>24 hours = 24 x 60 minutes = 1440 minutes 1440 ÷ 20 = 72 1 x 2⁷² = 4722366483000000000000 (rounded) = 4.72 x 10²¹ (see section on Standard Form)</p>																																															

2.2.4 Division

980 ÷ 4																												
Method 1	Method 2																											
Short division	Chunking																											
<p>This method is also known as the “bus stop.”</p> <p>Write the number you are dividing by outside the “bus stop”, and the other number under it.</p> <p>There are 2 fours in 9 with remainder 1 so the answer starts with 2 and the remainder 1 is placed next to the 8.</p> <p>There are 4 fours in 18 with remainder 2.</p> <p>There are 5 fours in 20 with no remainder.</p> $\begin{array}{r} 2 \quad 4 \quad 5 \\ 4 \overline{) 980} \\ \underline{8} \\ 18 \\ \underline{16} \\ 20 \\ \underline{20} \\ 0 \end{array}$ <p>The answer is 245</p>	<p>Since we are dividing by 4, we need to work out how many 4s are needed to make 980.</p> <p>We can do this by calculating multiples of 4 subtracting them from 980 until we get down to zero.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td>980</td> </tr> <tr> <td>100 x 4 =</td> <td>400</td> <td>980 - 400 = 580</td> </tr> <tr> <td>100 x 4 =</td> <td>400</td> <td>580 - 400 = 180</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>180 - 40 = 140</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>140 - 40 = 100</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>100 - 40 = 60</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>60 - 40 = 20</td> </tr> <tr> <td>5 x 4 =</td> <td>20</td> <td>20 - 20 = 0</td> </tr> <tr> <td>245 x 4 =</td> <td>980</td> <td></td> </tr> </table> <p>The answer is 245</p>			980	100 x 4 =	400	980 - 400 = 580	100 x 4 =	400	580 - 400 = 180	10 x 4 =	40	180 - 40 = 140	10 x 4 =	40	140 - 40 = 100	10 x 4 =	40	100 - 40 = 60	10 x 4 =	40	60 - 40 = 20	5 x 4 =	20	20 - 20 = 0	245 x 4 =	980	
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245 x 4 =	980																											

Example - DT



When spacing lights in a structure, a common mistake is to divide the length by the number of lights you want to place, rather than the number of spaces needed.

In the example on the left, 3 lights are to be placed, but 4 spaces are needed. Therefore if all the spaces are to be equal you would divide the length by 4 to see how far apart to space the lights.

If $L = 40$ cm, then each light would need to be 10 cm apart with a space of 10 cm at each end.

2.3 Number Properties

2.3.1 Types of number

Type	Examples	
Even numbers	2, 4, 6, 8, 10....	Even numbers are divisible by 2. They end in 2, 4, 6, 8 or 0.
Odd numbers	1, 3, 5, 7, 9, 11...	Odd numbers end in 1, 3, 5, 7 or 9
Whole numbers	0, 1, 2, 3, 4, 5...	Whole numbers cannot be negative.
Integers	... -4, -3, -2, -1, 0, 1, 2 ...	Integers include negative values.
Square numbers	1, 4, 9, 16, 25, 36...	A square number is the result of multiplying an integer by itself. e.g. $3^2 = 3 \times 3 = 9$, so 9 is a square number.
Multiples	Multiples of 3: 3, 6, 9, 12...	The multiples of a number are simply the number multiplied by any whole number.
Factors	Factors of 20: 1, 20, 4, 5, 2, 10	A factor is a number that divides exactly into another number.
Prime numbers	2, 3, 5, 7, 11, 13, 17 ...	Prime numbers have exactly 2 factors. The only factors of 17 are 1 and 17. So 17 is a prime number.

2.3.2 Place Value

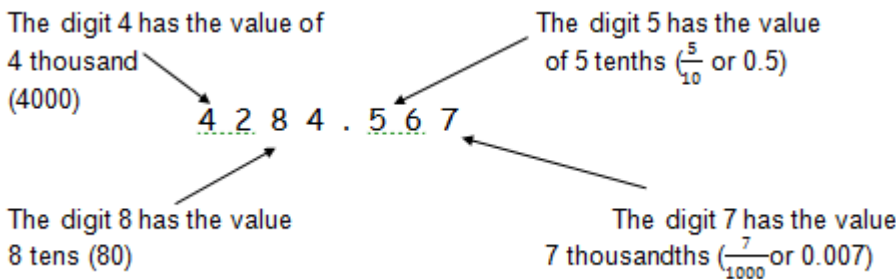
We use the decimal number system when doing calculations. Each digit has a place value. These place values are all powers of 10 and the main ones are shown in the table below.

	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
Power of 10	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value	1000s	100s	10s	1s	0,1s	0.01s	0.001s
					$\frac{1}{10}$ s	$\frac{1}{100}$ s	$\frac{1}{1000}$ s

Micro...	Nano...	Pico...
10^{-6}	10^{-9}	10^{-12}

These place values are often used in science.

For example 0.000007 m can be written as 7×10^{-6} m or **7 micrometers**.



There are 10 “thousandths” in a “hundredth”
 There are 10 “hundredths” in a “tenth”
 There are 10 “units” in a “ten”

There are 10 “tens” in a “hundred”
 There are 10 “hundreds” in a “thousand”

2.4 Standard Form

Key points	Examples - science
Standard form is a useful way to write very large or very small numbers.	The speed of light is approximately 300 000 000 m/s.
Numbers in standard form are written in the form: $A \times 10^n$	300 000 000 $= 3 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ $= 3 \times 10^8$ m/s
Where $1 \leq A < 10$ and n is an integer	The length of a virus is approximately 0.0000004 metres
If n is positive, the number will be larger than 1, if n is negative the number will be less than 1.	0.0000004
$\times 10^3$ means the same as $\times 10 \times 10 \times 10$	$= 4 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10$
$\times 10^{-4}$ means the same as $\div 10 \div 10 \div 10 \div 10$	$= 4 \times 10^{-7}$ m (=4 nanometres)

2.5 Other number bases (Computer science)

Binary	Hexadecimal																																																																
<p>The decimal system uses base 10. That means every time you reach the value 10 in a place value, instead of writing 10 you write zero and add one to the next place value up.</p> <p>You could, however, use any base.</p> <p>Binary uses base 2 instead. It means that using Binary all numbers can be written in terms of 0 and 1. This makes it particularly useful for computers to communicate in, as it simplifies all numbers to a string of 0s and 1s.</p> <table border="1" data-bbox="113 674 783 920"> <thead> <tr> <th>Decimal number</th> <th>Binary number</th> </tr> </thead> <tbody> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>10</td></tr> <tr><td>3</td><td>11</td></tr> <tr><td>4</td><td>100</td></tr> <tr><td>5</td><td>101</td></tr> <tr><td>6</td><td>111</td></tr> </tbody> </table> <p>Look at the first 8 place values in binary. $(2^7=128; 2^6=64; 2^5=32; 2^4=16; 2^3=8; 2^2=4; 2^1=2; 2^0=1)$</p> <table border="1" data-bbox="113 1043 791 1115"> <thead> <tr> <th>128s</th> <th>64s</th> <th>32s</th> <th>16s</th> <th>8s</th> <th>4s</th> <th>2s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>The number in bold is known as a “bit pattern” and it is written in binary.</p> <p>As a decimal it can be calculated as: $(1 \times 128) + (0 \times 64) + (0 \times 32) + (1 \times 16) + (1 \times 8) + (0 \times 4) + (0 \times 2) + (1 \times 1)$ = 153</p>	Decimal number	Binary number	1	1	2	10	3	11	4	100	5	101	6	111	128s	64s	32s	16s	8s	4s	2s	1s	1	0	0	1	1	0	0	1	<p>Hexadecimal uses base 16.</p> <p>A hex digit can be any of the following: 0 1 2 3 4 5 6 7 8 9 A B C D E F</p> <p>Each string of 4 binary digits can be represented by the hexadecimal system as follows:</p> <table border="1" data-bbox="852 573 1477 1171"> <thead> <tr> <th>Binary</th> <th>Hexadecimal</th> </tr> </thead> <tbody> <tr><td>0000</td><td>0</td></tr> <tr><td>0001</td><td>1</td></tr> <tr><td>0010</td><td>2</td></tr> <tr><td>0011</td><td>3</td></tr> <tr><td>0100</td><td>4</td></tr> <tr><td>0101</td><td>5</td></tr> <tr><td>0110</td><td>6</td></tr> <tr><td>0111</td><td>7</td></tr> <tr><td>1000</td><td>8</td></tr> <tr><td>1001</td><td>9</td></tr> <tr><td>1010</td><td>A</td></tr> <tr><td>1011</td><td>B</td></tr> <tr><td>1100</td><td>C</td></tr> <tr><td>1101</td><td>D</td></tr> <tr><td>1110</td><td>E</td></tr> <tr><td>1111</td><td>F</td></tr> </tbody> </table> <p>So the binary number 01111110 would be given as: 7 E</p>	Binary	Hexadecimal	0000	0	0001	1	0010	2	0011	3	0100	4	0101	5	0110	6	0111	7	1000	8	1001	9	1010	A	1011	B	1100	C	1101	D	1110	E	1111	F
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1101	D																																																																
1110	E																																																																
1111	F																																																																
<p>Binary shift / addition</p> <p>Since each place value in binary is simply twice the size of the place value to the right of it, it makes it easy to multiply and divide binary digits by powers of 2. This is known as “binary shift”. Multiply will shift the digits to the left, dividing will shift the digits to the right.</p> <p>Example <i>Multiply the following bit pattern by 8</i> 00011001 $8 = 2^3 = 2 \times 2 \times 2$</p> <p>Therefore shift all the digits 3 places to the left: 00011001 x 8 = 11001000</p> <p>(When adding binary numbers simply work out the total number of each power of 2 that you have.)</p>	<p>Example</p> <p>A bit pattern is given as: 01001110</p> <p>a) Convert the bit pattern into decimal. $(0 \times 128) + (1 \times 64) + (0 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1) = \mathbf{78}$</p> <p>b) Convert the bit pattern into hexadecimal. 0100 = 4 1110 = E 4E</p>																																																																

2.6 Estimation and Accuracy

2.6.1 Estimation

Students often struggle to make estimates of the everyday measures they are surrounded by.

In PE students find it difficult to judge a specific distance, for example a throw of 4 metres.

They can also struggle to understand what travelling at a given speed would feel like, for example 10 kilometres an hour.

Students also struggle with concepts such as judging half way distances, or a third of the way across a given space.



Key Points

Estimation allows us to get an approximate value for something.

You can estimate in two ways, but both involve a Mathematical approximation. Estimation is not guessing.

If you are asked to estimate in a calculation, you are expected to round the numbers in the calculation before carrying it out.

Example 1

I buy 11 packs of spaghetti. Each packet costs £1.99. Estimate the total cost.

£1.99 is approximately £2.

Total cost \approx £2 x 11 = £22

In mathematics we use the symbol " \approx " to show we are making an approximation or estimate.

In science the symbol " \approx " is usually used instead.

If you are asked to estimate something without numerical data being given, use heights, weights and volumes you are familiar with in everyday life to help you make an approximation for the object in question.

Practical ways to estimate

These facts can be used to help you estimate lengths, volumes and weights of objects.

A door is around **2 m** tall



A thumb nail is about **1 cm** across



A typical pencil weighs just under **10 g**.



A typical bag of sugar is either **500 g** or **1 kg**.



A small bottle of drink typically contains **250 ml**



A large bottle of drink often contains **2 litres**.



Example 2

What is an estimate for the height of the bus?



A typical man is just shorter than a door, so an estimate for a man's height could be 1.7 m. The bus is just over twice the height of the men. $2 \times 1.7 = 3.4$ m, so an estimate for the bus' height could be 3.7 m.

2.6.2 Accuracy and Rounding

Key points

To make numbers easier to use or read we often round them.

What we round them to depends on how accurate we want our answers to be.

The first way to round is to use place value.

e.g. Round 32457 to the **nearest hundred**

“4” is in the hundreds.

32457 to the nearest hundred will be either 32400 or 32500 depending on which of these it is nearer to.

To decide which it is nearer to look we look at the digit in the next place value, in this case in the tens.

32457

If the next digit is 5 or more we round up to the higher value, if it is 4 or less we round down to the lower value.

Therefore

32457 to the nearest hundred is
32500

e.g. Round 23564 to the **nearest ten**

23564 (the 4 means we round down, not up)
23560

e.g. Round 2465970 to the **nearest thousand**

2465970 (the 9 means we round up)
2466000

Decimal places refer to how many numbers are after the decimal point.

e.g. Round 23.86547 to **one decimal place**

23.86547 (the 6 means we round up)
23.9

N.B. You do not need to write zeros to hold the place values at the end of a number if the place values are after the decimal point.

e.g. Round 568.32456 to **two decimal places**

568.32456 (the 4 means we round down)
568.32

e.g. Round 348.978 to one decimal place

348.978

The 7 rounds up the 9 to 10, so add one to the place value to the left, the units)

349.0

Rounding to significant figures

The first significant figure (s.f.) in a number, is the first digit with any size.

The first significant figures in the following numbers are in bold. When rounding to one significant figure, you round to the place value that the first significant figure is in. Look at the next digit to see whether to round up or round down.

32152496 \approx 30000000 (to 1 s.f.)

235.60567 \approx 200 (to 1 s.f.)

0.000**3**56537 \approx 0.0004 (to 1 s.f.)

The second significant figure in a number, is the second digit with any size.

When rounding to two significant figures, you round to the place value that the second significant figure is in. Look at the next digit to see whether to round up or round down.

32152496 \approx 32000000 (to 2 s.f.)

235.60567 \approx 240 (to 2 s.f.)

0.000**35**6537 \approx 0.00036 (to 2 s.f.)

Example – Physics

[A] solar storage power station can store a maximum of 2 200 000 kWh of energy.

The solar storage power station can supply a town with a maximum electrical power of 140 000 kW.

Calculate for how many hours the energy stored by the solar storage power station can supply the town with electrical power.

Give your answer to 2 significant figures.

Use the correct equation from the Physics Equations Sheet.

$P = \frac{E}{t}$	P	power
	E	energy transferred
	t	time taken

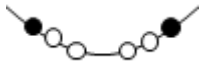

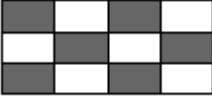
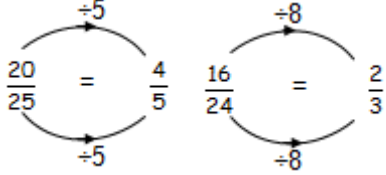
Rearranging:

$$t = \frac{E}{P} = \frac{2200000}{140000} = 15.71428 \dots$$

The second significant figure is the “5” in the units place value, therefore round to the nearest whole number. The “7” rounds the 15 up to 16.

15.71428 \approx **16 hours**

2.7 Fractions

Terminology	Equivalent fractions	Simplifying fractions
<p>Numerator → $\frac{3}{4}$</p> <p>Denominator →</p> <p>What fraction of the beads are black?</p>  <p>2 beads out of a total of 6 are black.</p> <p>$\frac{2}{6}$ of the beads are black.</p>	 $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ <p>These fractions all represent the same proportion of a whole. They are equivalent.</p>  $\frac{6}{12} = \frac{30}{60}$ <p style="text-align: center;">↻ x5 ↻</p>	<p>To simplify a fraction you divide the numerator and denominator by the same number, a common factor.</p>  <p>When simplifying fractions avoid having decimals in the fraction. If you are asked to put a fraction in its simplest form, the question wants you to simplify the fraction as much as possible.</p>

2.7.1 Finding fractions of amounts

The denominator of a fraction tells you how many equal parts the whole has been divided into. The numerator of a fraction tells you how many of those parts you have.

For example: Find $\frac{2}{5}$ of £150:

Divide £150 into 5 equal parts: $£150 \div 5 = £30$

Find the value of 2 of these parts: $£30 \times 2 = \underline{\underline{£60}}$


2.8 Percentages

The basics	
“%” means out of 100	100% means $\frac{100}{100}$ or the whole amount.
63% means $\frac{63}{100}$	Percentages can be more than 100, e.g. 120% Percentages do not have to be whole numbers e.g. 12.5%
To find what percentage one amount is of another:	
<ol style="list-style-type: none"> Write what fraction one quantity is of another Convert this to a percentage by finding this fraction of 100% 	<p>Example - RE</p> <p>In 2010 approximately 2.2 billion of the world’s 9 billion people were Christian. What percentage of the world population is this?</p> $\frac{2.2}{9} \times 100 = (100 \div 9) \times 2.2 = \mathbf{24.4\%}$

2.8.1 Finding percentages of Amounts

Method 1	Method 2	Method 3
Use equivalent fractions	Use equivalent fractions	Use 10%
Find the equivalent fraction Simplify it Find that fraction of the amount.	Find the equivalent fraction. Find that fraction of the amount.	$10\% = \frac{10}{100} = \frac{1}{10}$ Find 10% of the amount and then use this to find the required percentage.
Find 50% of 2000 kg $50\% = \frac{50}{100} = \frac{1}{2}$ $\frac{1}{2}$ of 2000 kg = $2000 \div 2$ = <u>1000 kg</u>	Find 9% of 200 W $9\% = \frac{9}{100}$ $\frac{9}{100}$ of 200 W = $(200 \div 100) \times 9$ = <u>18 W</u>	Find 70% of £35 10% of £35 = $\text{£}35 \div 10$ = £3.50 70% = 7 x 10% 7 x £3.50 = <u>£24.50</u>

2.8.2 Additional calculator method



There is a percentage button on calculators. In Mathematics lessons we ask students not to use it as we feel it discourages mathematical understanding. This is how it is used.

Find 64% of 400

6
4
SHIFT
%
×
4
0
0
=

2.8.3 Percentage change

Key Points	Example – Chemistry
<p>To find a percentage change:</p> <ol style="list-style-type: none"> 1. Calculate the change 2. Find this change as a percentage of the original amount. <p>Example</p> <p>A car is bought for £3200 and sold for £2400. What is the percentage loss?</p> <p>Loss = £3200 - £2400 = £800</p> <p>Percentage loss = $\frac{800}{3200} \times 100 = 25\%$</p>	<p>Percentage yield calculations are used in Chemistry to look at the products produced by a reaction in practice, compared to what you would expect theoretically.</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>Percentage yield = $\frac{\text{actual yield}}{\text{theoretical yield}} \times 100$</p> </div> <p>In the neutralisation of sulfuric acid with sodium hydroxide, the theoretical yield from 6.9g of sulfuric acid is 10g. In a synthesis, the actual yield is 7.2g. What is the percentage yield for this synthesis?</p> <p>Percentage yield = $\frac{7.2}{10} \times 100 = 72\%$</p>

2.8.4 Fraction, Decimal & Percentage Equivalence

The equivalence of the most frequently used fractions, decimals and percentages is summarised in the table below:

Fraction	Decimal	Percentage
1	1	100 %
$\frac{1}{2}$	0.5	50 %
$\frac{1}{3}$	0.333...	33 %
$\frac{1}{4}$	0.25	25 %
$\frac{3}{4}$	0.75	75 %
$\frac{1}{10}$	0.1	10 %
$\frac{2}{10} (= \frac{1}{5})$	0.2	20 %
$\frac{3}{10}$	0.3	30 %

Example - Biology

Genetic diagrams show how chromosomes, or alleles, coming from different parents combine to give the genetic characteristics of the child.

This genetic diagram shows how, if both parents carry the recessive allele "f" for cystic fibrosis, this can lead to them having a child who is homozygous (ff) for the recessive allele, and hence who will develop cystic fibrosis.

		Mother	
		F	f
Father	F	FF	Ff
	f	Ff	ff

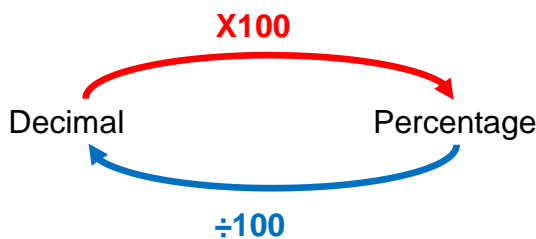
The chance of the child whose parents both carry the recessive allele inheriting cystic fibrosis from them is 1 in 4, or $\frac{1}{4}$ or 25%.

The chance of the child being "Ff" is 2 in 4, or $\frac{1}{2}$ or 50%.

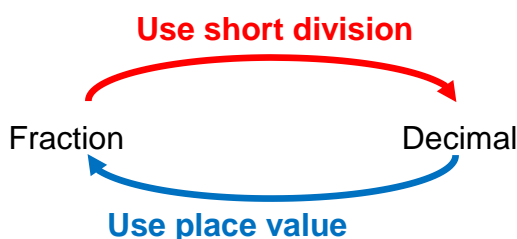
Knowledge of these equivalent fractions, decimals and percentages is useful in both DT and Science.

2.8.4.1 Converting between fractions, decimals and percentages

The following give methods to convert between fractions, decimals and percentages.



Convert it to a decimal, then multiply by 100



Examples

Decimal to percentage
 $0.36 = 0.36 \times 100\% = 36\%$

Percentage to decimal
 $4\% = 4 \div 100 = 0.04$

Percentage to fraction
 $56\% = \frac{56}{100}$

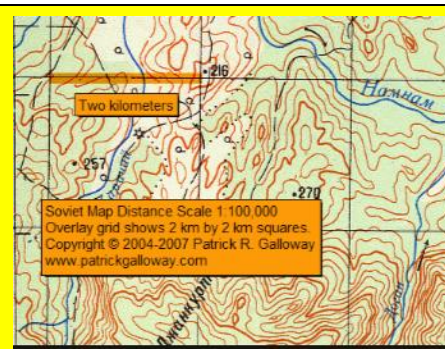
Fraction to percentage
 $\frac{2}{5} = 0.4 = 0.4 \times 100\% = 40\%$

Fraction to decimal
 $\frac{3}{4} = \frac{0.75}{1} = 0.75$

Decimal to fraction
 $0.4 = \frac{4}{10}$

2.9 Ratio & Proportion

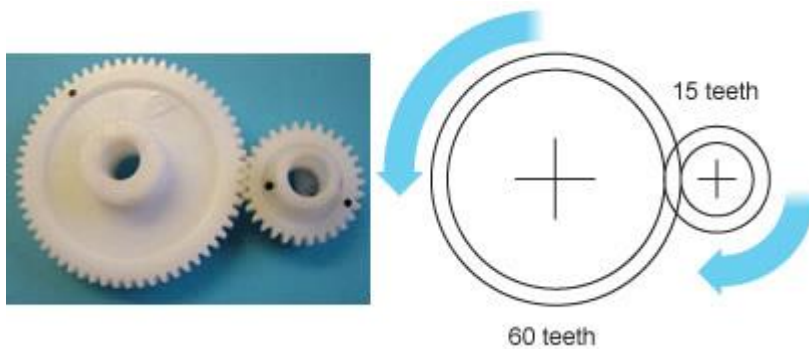
2.9.1 Ratio

Writing a ratio	Ratios as fractions	Ratios and proportion
<p>A ratio tells you how much you have of one thing compared to another.</p> <p>To make pastry you may need to mix 2 parts flour to 1 part fat. This means the ratio of flour to fat is 2:1.</p> <p>The order of the numbers in the ratio is important.</p>	<p>The ratio of flour to fat is 2 : 1.</p> <p>2 parts + 1 part = 3 parts total</p> <p>$\frac{2}{3}$ of the mixture is flour</p> <p>$\frac{1}{3}$ of the mixture is fat</p>	<p>The ratio of flour to fat is 2 : 1</p> <p>Let the amount of flour = a</p> <p>Let the amount of fat = b</p> <p>There is twice as much flour as fat therefore if the ratio of a to b is 2 : 1,</p> <p>$a = 2b$</p>
Simplifying ratios	Ratio and scale	
<p>Ratios are simplified in a similar way to fractions, find a common factor and divide each part by that number.</p> <p>Simplify 5 : 35 : 20</p> <p>Divide each part by 5</p> <p>1 : 7 : 4</p>	 <p>Ratio is used to show the relationship between a distance on a map, and the actual distance in reality.</p> <p>A scale of 1:100 000 on a map means that 1 cm on the map represents 100 000 cm (= 1 km) in reality.</p>	

Applications to reacting mass calculations	Applications to balancing equations
<p>In all chemical reactions, the total mass of reactants used is equal to the total mass of the products made</p> <p>For any one reaction, the ratio of reactant to product does not change.</p> <p>What mass of carbon dioxide is formed when 15 g of carbon is burned in air?</p> <p>$C + O_2 \rightarrow CO_2$</p> <p>Work out the relative masses of the substances needed in the calculation.</p> <p>Mass of carbon = 12, Mass of carbon dioxide = 44</p> <p>Mass of C : Mass of CO_2</p> <p>$\div 12$ $\begin{matrix} 12 : 44 \\ 1 : 3.67 \end{matrix}$ $\div 12$</p> <p>15 g x 3.67 = 55.05 g</p>	<p>Chemical equations need to be balanced. The number of each type of atom on each side of the equation must be the same</p> <p>To make things equal, you need to adjust the number of units of some of the substances until you get equal numbers of each type of atom on both sides of the arrow.</p> <p>Copper + Oxygen \rightarrow Copper (II) Oxide</p> <p>Unbalanced equation:</p> <p>$Cu + O_2 \rightarrow CuO$</p> <p>Balanced equation:</p> <p>$2Cu + O_2 \rightarrow 2CuO$</p> <p>Ratio of Copper atoms to Copper (II) Oxide molecules produced is 2 : 2 which simplifies to 1 : 1.</p>

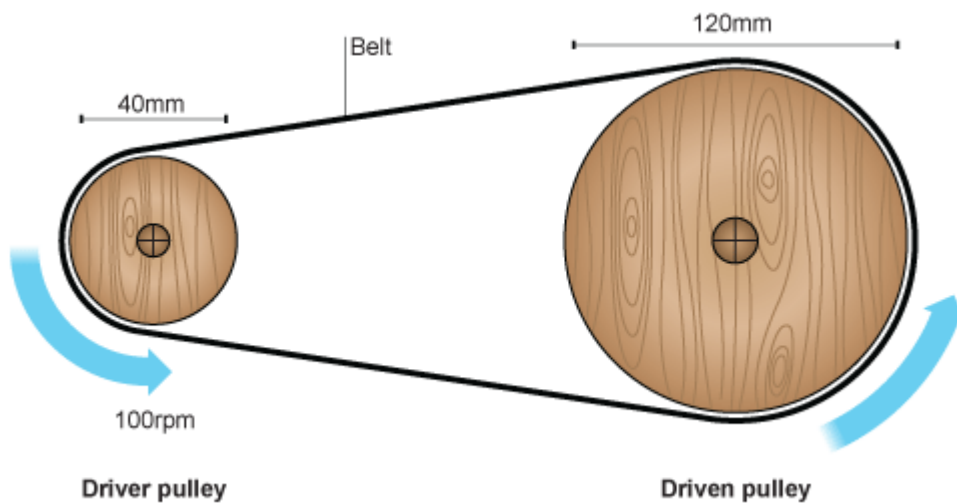
Gear and Velocity Ratios – Design and Technology

Gears are used in machinery to transmit rotary motion from one part of the machine to another. When gears are connected smaller gears rotate faster than larger gears. The relationship between the speeds the gears rotate at is called the gear ratio (or sometimes velocity ratio).



$$\text{Gear ratio} = \frac{\text{number of teeth on driven gear}}{\text{number of teeth on driver gear}} = \frac{60}{15} = 4$$

Pulleys are also used in machinery to transmit rotary motion. A pulley system consists of two pulley wheels each on a shaft, connected by a belt.



If the pulley wheels are different sizes, the smaller one will spin faster than the larger one. The difference in speed is called the velocity ratio. This is calculated using the formula:

$$\text{Velocity ratio} = \frac{\text{diameter of driven pulley}}{\text{diameter of driver pulley}}$$

$$\text{Velocity ratio} = \frac{120}{40} = 3$$

2.9.2 Proportion

Direct proportion

If the ratio between two quantities is constant, they are said to be in direct proportion.

5.85 g grams of sodium chloride are produced when 5.3 g of sodium carbonate reacts with dilute hydrochloric acid.

How many grams of sodium chloride would be produced if 15.9 g of sodium carbonate was reacted with dilute hydrochloric acid?

Sodium carbonate	Sodium chloride
5.3 g	5.85 g
15.9 g	17.55 g

(Note: Red arrows in the original image show a multiplier of x3 from 5.3 to 15.9 and from 5.85 to 17.55.)

The scale on a map is 1 : 50 000

The distance between two landmarks on a map is measured as 2.5 cm, what actual distance does this represent?

Distance on map	Actual distance
1 cm	50 000 cm
2.5 cm	125 000 cm (=1.25 km)

(Note: Red arrows in the original image show a multiplier of x2.5 from 1 cm to 2.5 cm and from 50 000 cm to 125 000 cm.)

The ingredients to make 8 scones are shown in the table below, how much of each ingredient would be needed to make 10 scones?

	8 scones	1 scone	10 scones
Self-raising flour	350 g	43.75 g	437.5 g
Baking powder	1 tsp	$\frac{1}{8}$ tsp	$\frac{10}{8}$ tsp = $1\frac{1}{4}$ tsp
Butter	85 g	10.625 g	106.25 g
Caster sugar	3 tbsp	$\frac{3}{8}$ tbsp	$\frac{30}{8}$ tbsp = $3\frac{3}{4}$ tbsp
Milk	175 ml	21.875 g	218.75 g

(Note: Red arrows in the original image show a multiplier of ÷8 from 8 scones to 1 scone, and a multiplier of x10 from 1 scone to 10 scones.)

Inverse proportion

Inverse proportion is a relationship where as one quantity increases, the other decreases.

Example – Physics – Inverse square law

Photosynthesis uses energy from light. The rate of photosynthesis can be increased by increasing the light intensity. Light intensity itself is affected by how far the plant is from the source of light.

The intensity of light at different distances from a light source can be described by the inverse square law. This states that the intensity of light is inversely proportional to the square of the distance from the source.

Light intensity can be calculated using this formula

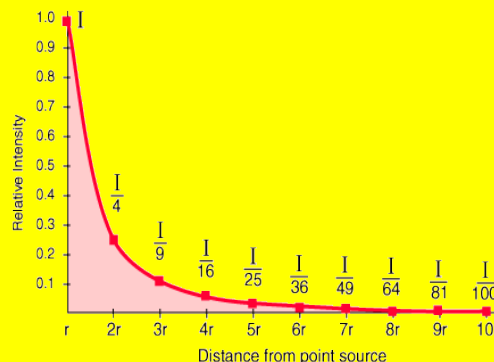
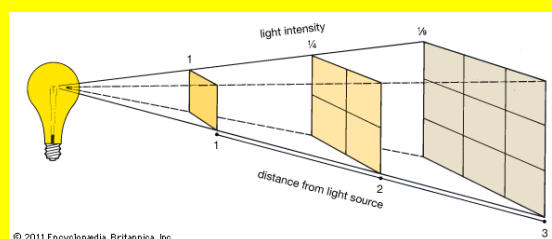
$$\text{light intensity} \propto \frac{1}{\text{distance}^2}$$

When a light source is 25 cm from a plant, it will receive

$$\frac{1}{0.25^2} = 100 \text{ arbitrary units}$$

If the light source is 50 cm from the plant (double the distance), it will only receive a quarter as much light.

$$\frac{1}{0.5^2} = 4 \text{ arbitrary units}$$

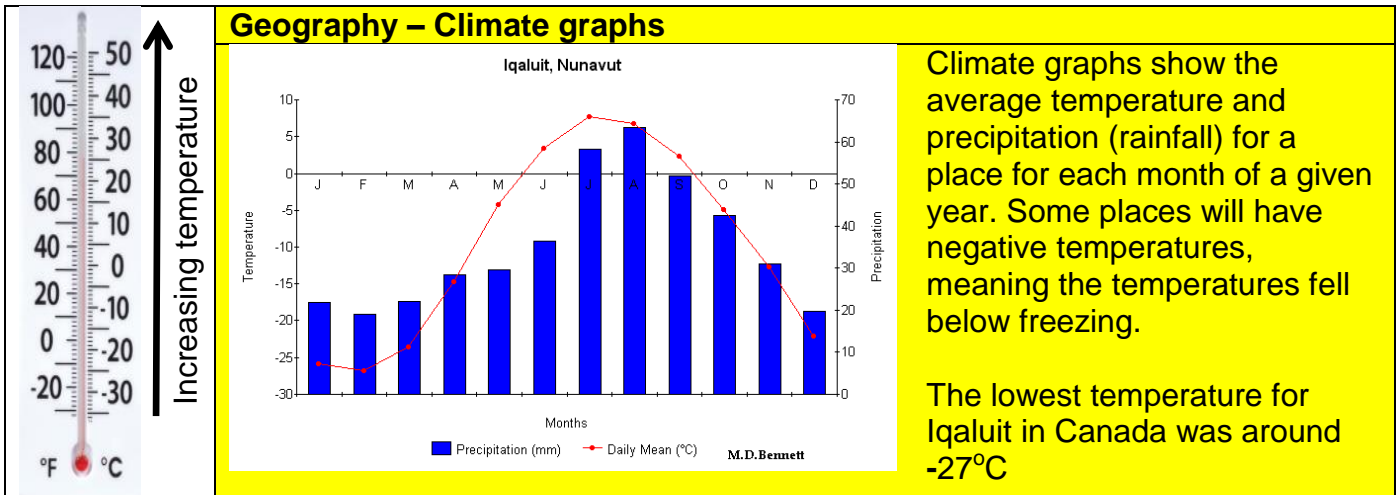


2.10 Directed Numbers

According to teachers, the concept of directed numbers is one that students struggle with across the curriculum. Directed numbers are numbers that are given a sign, either positive or negative.

A negative number is a number less than zero. You can tell a number is negative if it has a minus (-) sign in front of it. The more negative a number is, the smaller it is. For example -8 is smaller than -3 .

2.10.1 Temperature



2.10.3 Multiplying and dividing negative numbers

If there is no sign in front of a number, it is positive.

$$5 \times 7 = 35 \quad -5 \times 7 = -35 \quad 5 \times -7 = -35 \quad -5 \times -7 = 35$$

$$48 \div 6 = 8 \quad -48 \div 6 = -8 \quad 48 \div -6 = -8 \quad -48 \div -6 = 8$$

When multiplying and dividing numbers, if both numbers are positive OR if both numbers are negative, you will get a positive answer. If only one of the numbers is negative you will get a negative answer.

2.11 Coordinates

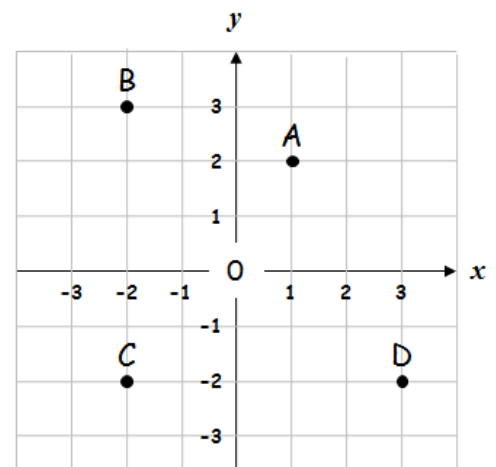
Cartesian coordinates

Coordinates are used to describe location. In Mathematics we use the Cartesian coordinate system to describe location in the $x - y$ plane. Coordinates are given as two numbers in a bracket separated by a comma.

The first number is the x -coordinate (how far you need to travel in the x -direction) and the second number is the y -coordinate (how far you need to travel in the y -direction).

The coordinates of the points shown are:
A (1, 2) ; B (-2, 3) ; C (-2, -2) and D (3, -2)

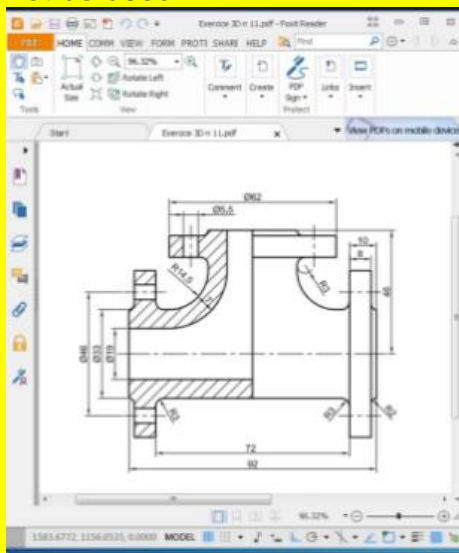
The point (0, 0) is known as the "origin."



Laser cutters

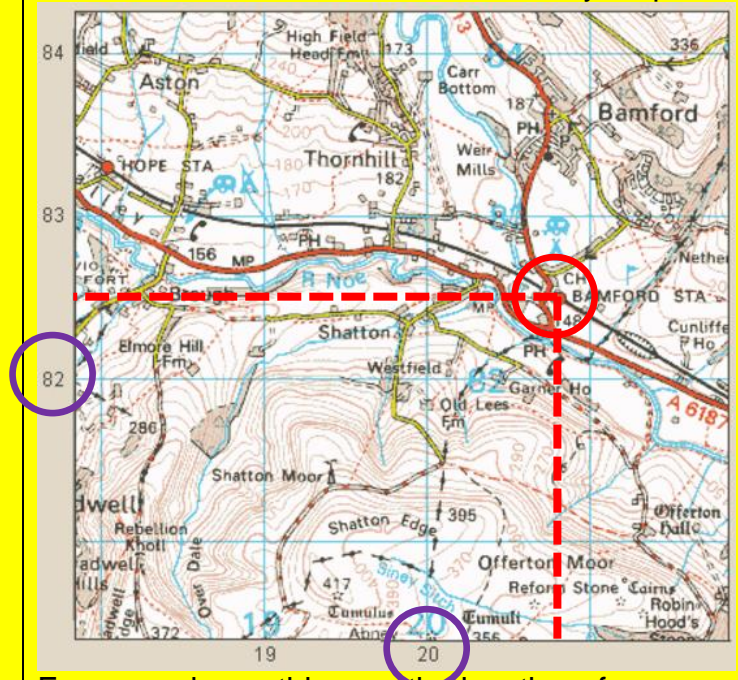
In DT, laser cutters use the coordinates that are programmed into them to produce designs.

The bottom left corner of the design should have coordinates as close to (0, 0) as possible and negative coordinates should not be used.



Ordinance survey coordinates

A four or six figure grid reference can be used to show a location on an ordinance survey map.



For example, on this map the location of "Bamford train station" could be given as "2082" or as "208825"

2.12 Inequalities

Inequalities are used to show whether one value is greater or less than another.

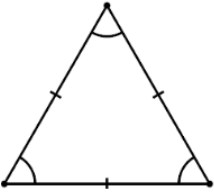
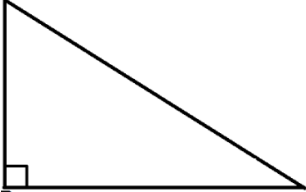
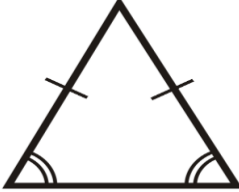
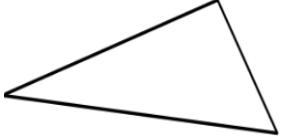
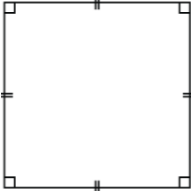
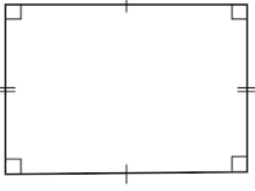
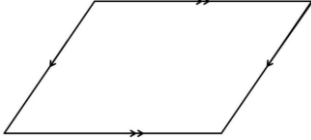
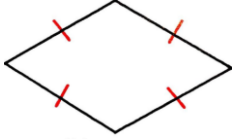

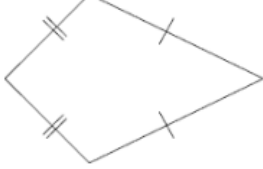

	Meaning		Meaning
<	“less than”	diameter < 10mm	The diameter is less than 10 mm
≤	“less than or equal to”	height ≤ 5 m	The height is less than or equal to 5 m
>	“greater than”	mass > 10 g	The mass is greater than 10 g
≥	“greater than or equal to”	length ≥ 40 cm	The length is greater than or equal to 40 cm

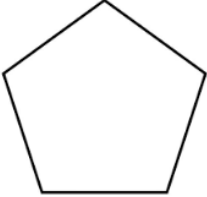
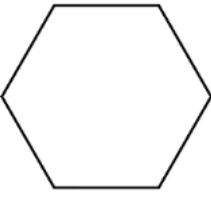
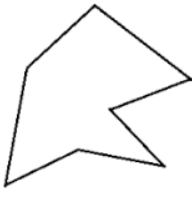
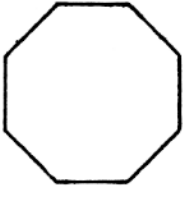
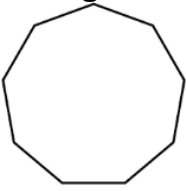
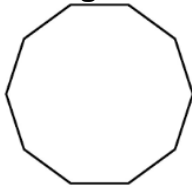
2.13 Shapes

2.13.1 Two dimensional (2D) shapes

A polygon is a closed 2D shape with straight sides.

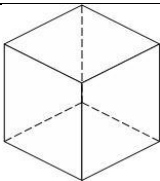
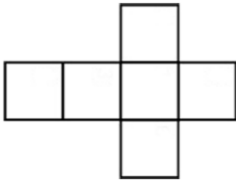
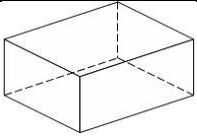
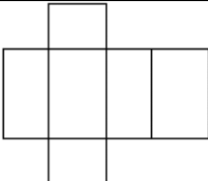
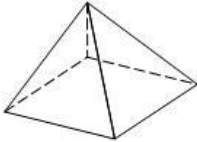
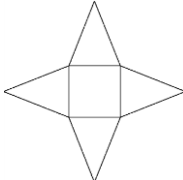
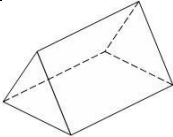
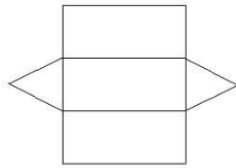

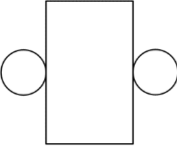
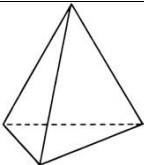
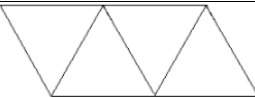
A regular polygon is a polygon where all the sides are equal length and all angles are the same size.

Types of triangles – 3 sided polygons			
Equilateral triangle  3 equal sides	Right-angled triangle  One angle is 90°	Isosceles triangle  Two equal sides	Scalene triangle  Each side is a different length
Types of quadrilaterals – 4 sided polygons			
Square  4 equal length sides 4 right angles (90°)	Rectangle  Opposite sides equal in length. 4 right angles (90°)	Parallelogram  Opposite sides equal in length. Opposite sides parallel.	Rhombus  4 equal length sides Opposite sides are parallel.
Trapezium  Two parallel sides.	Kite  Two pairs of equal length sides. One pair of equal angles.	Isosceles trapezium  Two parallel sides One pair of equal sides	

Pentagon  5 sided polygon	Hexagon  6 sided polygon	Heptagon  7 sided polygon	Octagon  8 sided polygon
Nonagon  9 sided polygon	Decagon  10 sided polygon		

2.13.2 Three dimensional (3D) shapes

3D shapes have length, width and height.

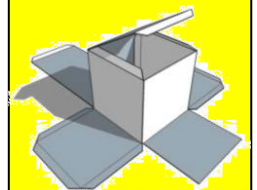
Shape	Name	Faces	Edges	Vertices	Net
	Cube	6	12	8	
	Cuboid	6	12	8	
	Square based pyramid	5	8	5	
	Triangular prism	5	9	6	
	Cylinder	3	2	0	
	Tetrahedron (Triangular based pyramid)	4	6	4	

A vertex is the correct mathematical name for a corner.

The plural of vertex is vertices.


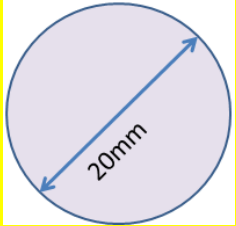
A net is a flat pattern that could be folded up to make a 3D shape.

Nets are used in Product Design in DT to create 3D structures and packaging.




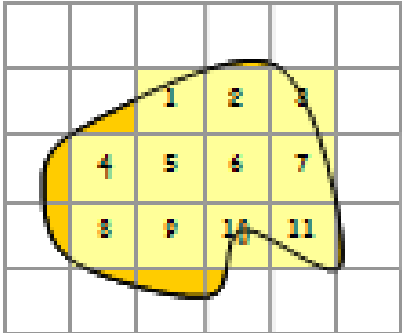


2.14 Area, perimeter and volume

2.14.1 Perimeter

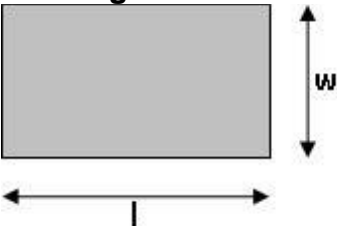
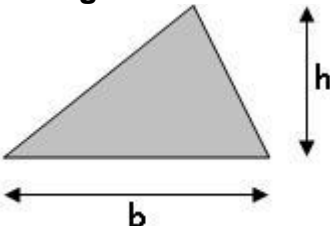
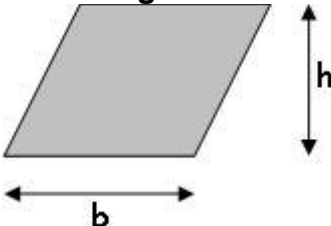
Definition	Example	Circumference of a circle (Used in DT)
<p>Perimeter is the distance round the outside of a shape.</p> <p>It is a length, and is therefore measured in units of length e.g. millimetres (mm), centimetres (cm) and metres (m).</p>	<p>Find the perimeter:</p>  <p>The shape has 4 sides, so to find the perimeter you need to add 4 lengths.</p> <p>Perimeter = $12 + 5 + 12 + 5$</p> <p style="text-align: center;">= <u>34 cm</u></p>	<p>The perimeter of a circle is known as its circumference and is calculated using:</p> <p>Circumference = $\pi \times \text{diameter}$</p> <p>Diameter : the distance across the circle through the centre</p> <p>π : The number 3.14...</p> <p>What is the circumference of this circle?</p> <p>$C = \pi \times 20 \text{ mm}$</p> <p style="text-align: center;">= <u>62.8 mm</u></p>  <p>Calculating the circumference is particularly useful when working out the length of rectangular sheet needed to wrap round the circular ends of a cylinder.</p>

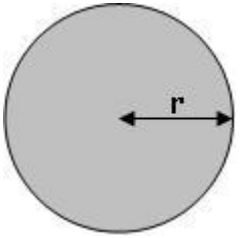
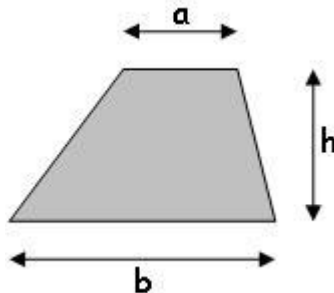
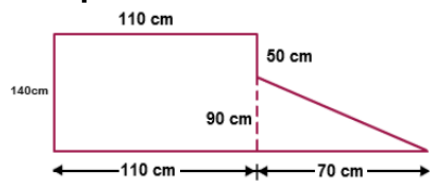
2.14.2 Area of 2D Shapes

Definition	Areas of irregular shapes
<p>The area of a shape is how much surface it covers.</p> <p>We measure area in square units e.g. centimetres squared (cm^2) or metres squared (m^2).</p>	<p>Given an irregular shape, we estimate its area through drawing a grid and counting the squares that cover the shape.</p> <div style="display: flex; align-items: flex-start;"> <div style="margin-right: 20px;"> <p> Whole square – count as one</p> <p> Half square or more – count as one</p> <p> Less than half a square - ignore</p> </div> <div style="text-align: center;">  </div> </div> <p>Area = <u>11</u></p> <p>N.B. This is an approximate value for the area, not the actual area.</p>

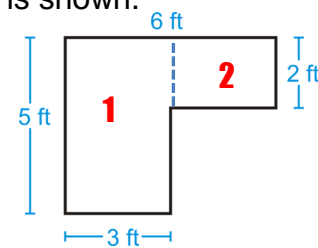
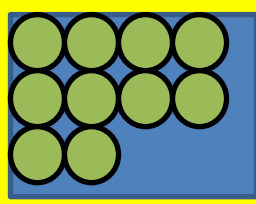
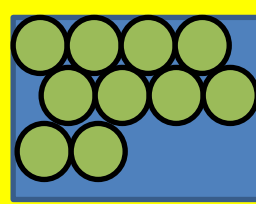
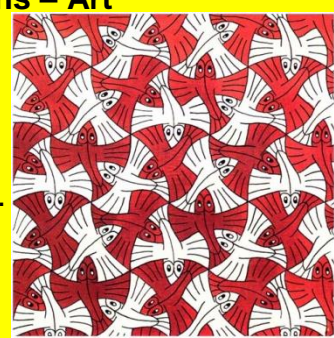
Area formulae

For common 2D shapes we can use formulae to calculate their exact area.

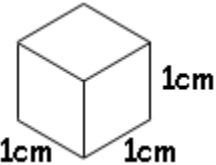
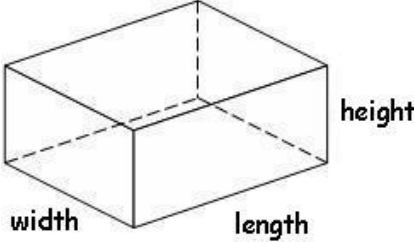
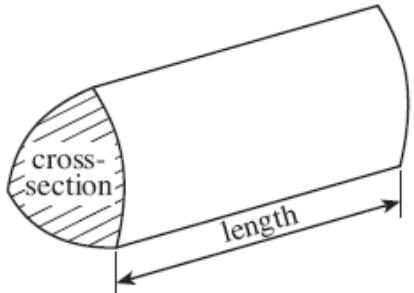
<p>Rectangle</p>  <p>Area = length x width</p>	<p>Triangle</p>  <p>Area = $\frac{1}{2}$ base x height</p>	<p>Parallelogram</p>  <p>Area = base x height</p>
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<p>Circle</p>  <p>$r = \text{radius} \quad \pi = 3.14\dots$</p> <p>$\text{Area} = \pi \times r \times r = \pi r^2$</p>	<p>Trapezium</p>  <p>$\text{Area} = \frac{1}{2} \times (a + b) \times h$</p>	<p>Compound areas</p>  <p>When finding the area of a shape you are not familiar with, simply divide it into shapes you can find the area of, and find the total of their separate areas.</p>
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Area calculations across the curriculum

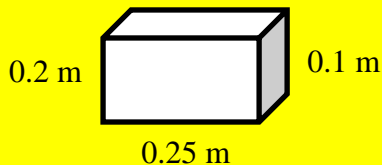
<p>Key points</p> <p>There are different kinds of area problems, each one is unique and it is important to think about each problem on a practical level.</p> <p>Some area problems require students to think about the total area, other problems require students to think about fitting specific shapes into a given area.</p>	<p>Example 2</p> <p>A diagram of a garden is shown.</p>  <p>The garden is going to be covered in grass. Each bag of grass seed covers 5 square feet (sqft) and costs £4.50.</p> <p>How much will the grass seed cost to cover the garden?</p> <p>Split the garden into areas we can calculate:</p> <p>Total area = Area 1 + Area 2 $= (5 \times 3) + (2 \times 3) = 15 + 6 = 21 \text{ sqft}$</p> <p>Calculate the numbers of bags of grass seed needed:</p> <p>$21 \div 5 = 4.2$ therefore we need 5 bags</p> <p>Calculate the cost:</p> <p>$5 \times \text{£}4.50 = \text{£}22.50$</p>
<p>Example 1 - DT</p> <p>A sheet of acrylic measures 3300mm x 2450mm. How many circles of diameter 50mm can you cut out of it?</p> <p>One way to cut out the circles out is shown:</p>  <p>The distance across each circle is 50 mm.</p> <p>Going across you could fit:</p> <p>$3300 \div 50 = 66 \text{ circles}$</p> <p>Going down you could fit:</p> <p>$2450 \div 50 = 49 \text{ circles}$</p> <p>In total: $66 \times 49 = \text{3234 circles}$</p> <p>However there are other ways to cut out the circles that could generate less waste material.</p> <p>Here is another example of how the circles could be cut out.</p>  <p>It is worth noting that laser cutters usually require shapes to have a clearance of at least 1 mm when they are being cut out.</p> <p>To get the best fit for cutting out shapes you may need to rotate them and reflect them.</p>	<p>Tessellations</p> <p>A tessellating pattern is a repeating pattern made of shapes fitted together while leaving no gaps or overlaps.</p> <p>Escher Tessellations – Art</p>  <p>Escher was a famous artist and Mathematician who explored different tessellating patterns.</p> <p>Escher used mathematics to create shapes that would not only tessellate, but would also represent familiar objects or animals.</p>

2.14.3 Volume

Definition	Volume of a cuboid	Volume of a prism / cylinder
<p>Volume is the amount of space that an object contains or takes up. The object can be a solid, liquid or gas.</p> <p>Volume is measured in cubic units e.g. cubic centimetres (cm³) and cubic metres (m³).</p> <p>This cube has a volume of 1 cm³</p> 	 <p>Volume = length x width x height</p>	<p>Prisms have a uniform cross section.</p>  <p>Volume = area of cross section x length</p> <p>Examples of prisms:</p> <ul style="list-style-type: none"> Triangular prisms Cuboids (Cylinders)

Calculations with density - Chemistry

A block of copper has the dimensions shown.



The density of copper is $8.96 \times 10^3 \text{ kg/m}^3$.

Calculate the mass of the copper block.

Mass = Density x Volume

Volume = $0.2 \times 0.25 \times 0.1 = 0.005 \text{ m}^3$

Mass = $8.96 \times 10^3 \times 0.005 = \mathbf{44.8 \text{ kg}}$



Surface area to volume ratios - Biology

The surface area of an object is a measure of the area covering the outside of it. The volume of an object is a measure of the space inside it.

A small object generally has a large surface area to volume ratio, while a bigger object has a smaller surface area to volume ratio.

As a cell grows its surface area to volume ratio decreases, making diffusion less efficient and ultimately stopping the cell growing.

To combat this cells can develop shapes that maximise their surface area, or once they get to a certain size split in two to create two cells with a higher surface area to volume ratio.

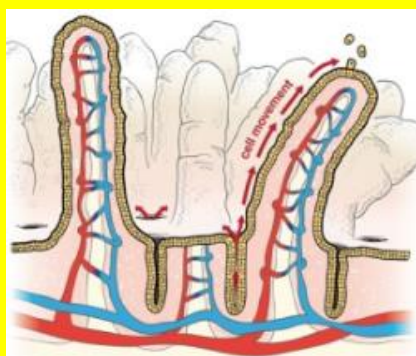


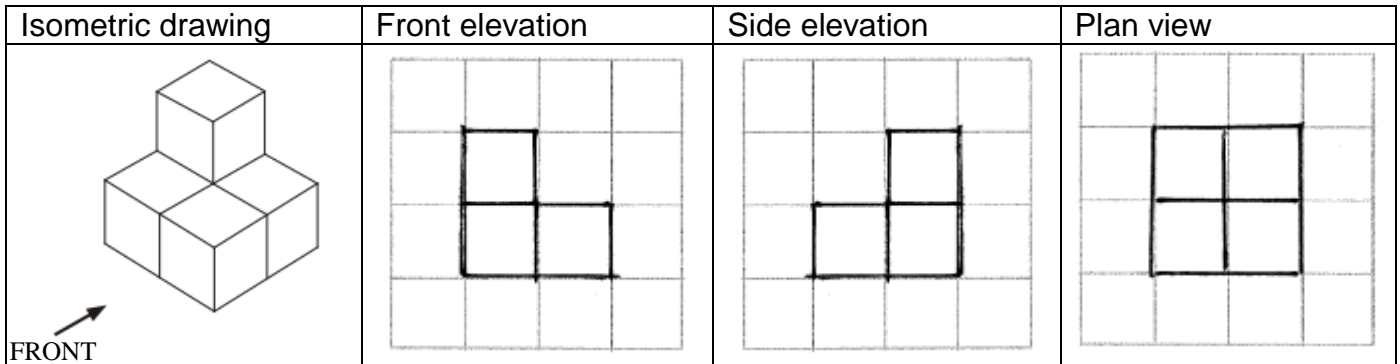
Diagram of villi – cells that have adapted to have a greater surface area in order to increase rates of diffusion.

2.14.4 Isometric drawings, plans and elevations

3D objects can be represented in Mathematics using either an isometric drawing (3D representation) or as a set of three 2D views known as a plan view, a front elevation and a side elevation. An arrow on the 3D view shows which direction is “front”.

The plan view is the view from above – often referred to as a “bird’s eye view”.

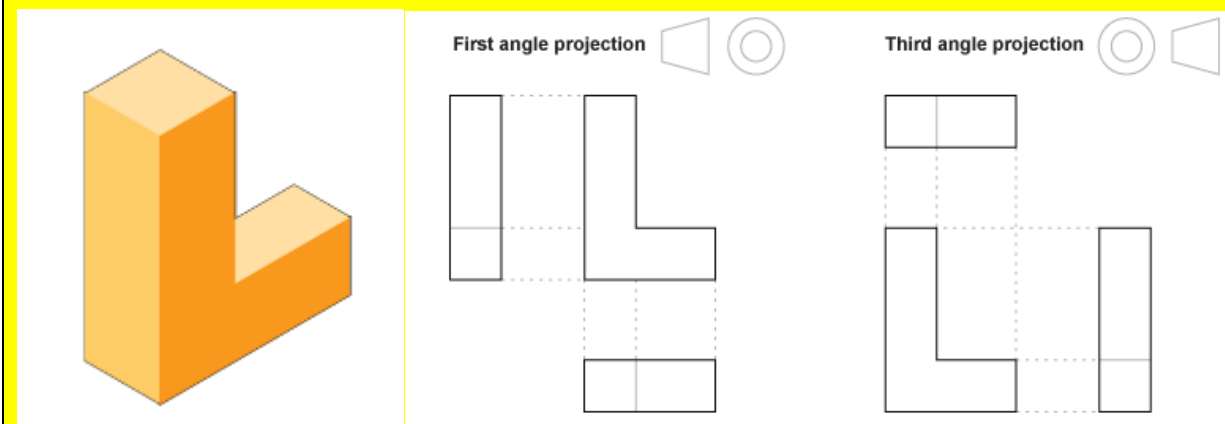
The front and side elevations are the views from the front and side respectively.



Orthographic projections – Design and technology

Orthographic drawings are used in Product Design to provide working drawings so that products can be manufactured. They usually consist of the same three views that are encountered in Mathematics - a front view, a side view and a plan, but sometimes more views are given to provide additional detail.

Orthographic drawing may be done using first angle projection or third angle projection.



2.15 Units of Measure

2.15.1 Metric units

For most measurements we use metric units. These are based on relationships connected to powers of 10 and are the units students will generally use across the curriculum.

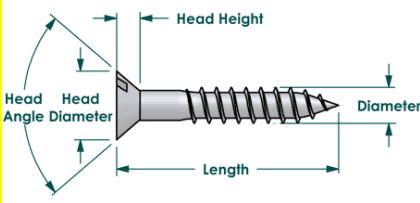
Length	Mass	Capacity (Volume)
1 km = 1000 m 1 m = 100 cm 1 cm = 10 mm km : kilometre cm : centimetre mm : millimetre A typical ruler used in lessons is either 15 cm or 30 cm long.	1 tonne = 1000 kg 1 kg = 1000 g 1 g = 1000 mg kg : kilogram g : gram mg : milligram A large bag of sugar weighs 1 kg.	1 l = 1000 ml 1 l = 10 cl 1 cl = 100 ml 1 ml = 1 cm ³ 1 l = 1 dm ³ l : litre ml : millilitre cl : centilitre A typical small bottle of drink contains 250 ml.

2.15.2 Compound measures

Compound measures involve a combination of units. They can be written in different forms. Some of the common ones are listed below.


	In words	Units	Units using indices - Science
Speed	metres per second	m/s	ms ⁻¹
Acceleration	metres per second squared	m/s ² or m/s/s	ms ⁻²
Density	grams per centimetre cubed	g/cm ³	gcm ⁻³
Concentration	grams per decimetre cubed	g/dm ³	gdm ⁻³

2.15.3 Converting between metric units

In general	In DT																												
<p>Metric units are directly proportional to each other. You can therefore use the relationships summarised in the table above to convert one metric unit to another.</p> <p>Convert 5.6 tonnes to kilograms</p> <table border="1"> <tr> <td>$\times 5.6$</td> <td>1 tonne</td> <td>1000 kg</td> <td>$\times 5.6$</td> </tr> <tr> <td></td> <td>5.6 tonnes</td> <td>5600 kg</td> <td></td> </tr> </table> <p>Convert 975 millilitres to litres</p> <table border="1"> <tr> <td>$\div 1000$</td> <td>1000 millilitres</td> <td>1 litre</td> <td>$\div 1000$</td> </tr> <tr> <td></td> <td>1 millilitre</td> <td>0.001 litres</td> <td></td> </tr> <tr> <td>$\times 975$</td> <td>975 millilitres</td> <td>0.975 litres</td> <td>$\times 975$</td> </tr> </table>	$\times 5.6$	1 tonne	1000 kg	$\times 5.6$		5.6 tonnes	5600 kg		$\div 1000$	1000 millilitres	1 litre	$\div 1000$		1 millilitre	0.001 litres		$\times 975$	975 millilitres	0.975 litres	$\times 975$	<p>In industry, for example engineering, measurements are often given in millimetres. You will therefore in DT be expected to convert measurements in centimetres to measurements in millimetres.</p>  <p>The length of a screw is measured as 3.5 cm. What is this measurement in millimetres?</p> <table border="1"> <tr> <td>$\times 3.5$</td> <td>1 cm</td> <td>10 mm</td> <td>$\times 3.5$</td> </tr> <tr> <td></td> <td>3.5 cm</td> <td>35 mm</td> <td></td> </tr> </table>	$\times 3.5$	1 cm	10 mm	$\times 3.5$		3.5 cm	35 mm	
$\times 5.6$	1 tonne	1000 kg	$\times 5.6$																										
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$\times 3.5$	1 cm	10 mm	$\times 3.5$																										
	3.5 cm	35 mm																											

2.15.4 Imperial units

Before metric units were introduced, we used imperial units to measure quantities. The relationships between imperial units vary more. Imperial units are still used in some subject areas, for example in DT when measuring quantities of food in recipes. In everyday life we still use miles to measure distances and stone to measure human mass (weight).

Length	Mass	Capacity
1 mile = 1760 yards	1 stone = 14 pounds (lbs)	1 gallon = 8 pints
1 yard = 3 feet	1 pound = 16 ounces (oz)	
1 foot = 12 inches		

2.15.5 Converting between imperial and metric units

We can convert between metric and imperial units using the following approximate relationships

Length	Mass	Capacity
1 inch \approx 2.5 cm	1 kg \approx 2.2 pounds	1 gallon \approx 4.5 litres
1 foot \approx 30 cm		1 pint \approx 0.6 litres
1 mile \approx 1.6 km		1 litre \approx 1.75 pints
5 miles \approx 8 km		

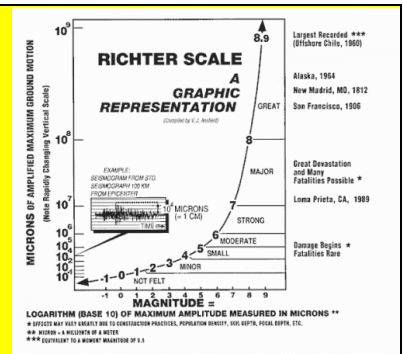
2.15.6 The Richter Scale – Geography

The magnitude or size of an earthquake can be measured by an instrument called a seismometer and shown on a seismograph.

Earthquakes are measured on the Richter scale from a value of 1 to 10.

Each level of magnitude is 10 times more powerful than the previous.

This type of scale is a logarithmic scale.



2.15.7 Units used in Computer Science

Measuring data

Students are expected to know the following conversions:

8 bits =	1 byte
1000 bytes =	1 kilobyte (kB)
1000 kilobytes =	1 megabyte (MB)
1000 megabytes =	1 gigabyte (GB)
1000 gigabytes =	1 terabyte (TB)



Example 1

Bob purchases a 4GB SD card for use as secondary storage in his phone. Calculate how many megabytes there are in 4GB.

$$4 \times 1000 = 4000 \text{ megabytes}$$

Example 2

A sound file has a size of 24,000 bits. What is 24,000 bits in kilobytes?

First convert 24000 bits to bytes
8 bits = 1 byte

$$24000 \text{ bits} = 24000 \div 8 \text{ bytes} = 3000 \text{ bytes}$$

Now convert 3000 bytes to kilobytes

$$3000 \text{ bytes} = 3000 \div 1000 \text{ kilobytes} = 3 \text{ kilobytes}$$

2.15.8 Units of time

1 millennium	=	1000 years
1 century	=	100 years
1 decade	=	10 years
1 year	=	365 days
1 leap year	=	366 days
1 year	=	12 months
1 year	=	52 weeks
1 week	=	7 days
1 day	=	24 hours
1 hour	=	60 minutes
1 minute	=	60 seconds

Season	Month	Days
Winter	January	31
	February	28 (or 29)
Spring	March	31
	April	30
	May	31
Summer	June	30
	July	31
	August	31
Autumn	September	30
	October	31
	November	30
Winter	December	31

History

In History you will talk about what century events occurred in.

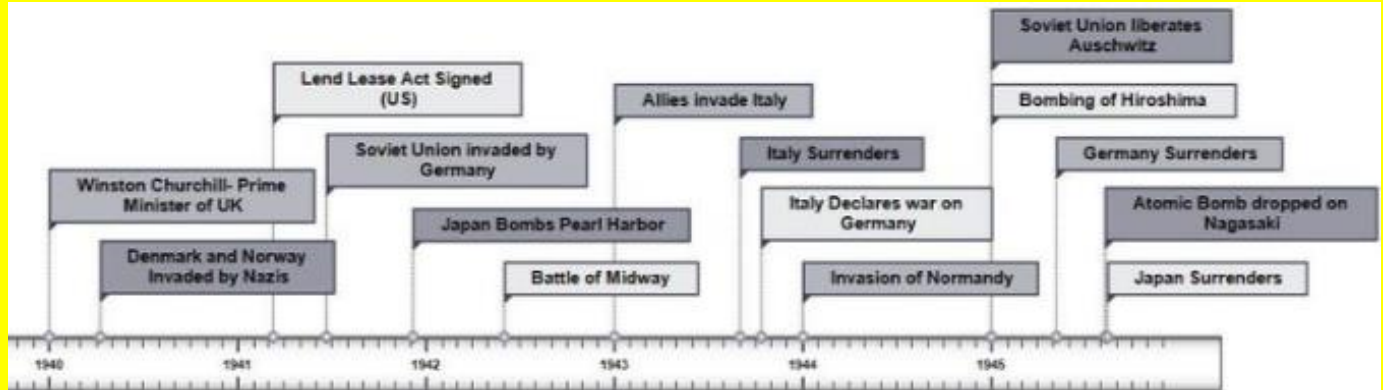
The 20th Century refers to 1900 – 1999

We are currently in the 21st Century.

Timelines

Timelines are used in History to show the sequence of events over time. They can be very useful in understanding how different events in History are connected.

The timeline below shows the events of World War II.



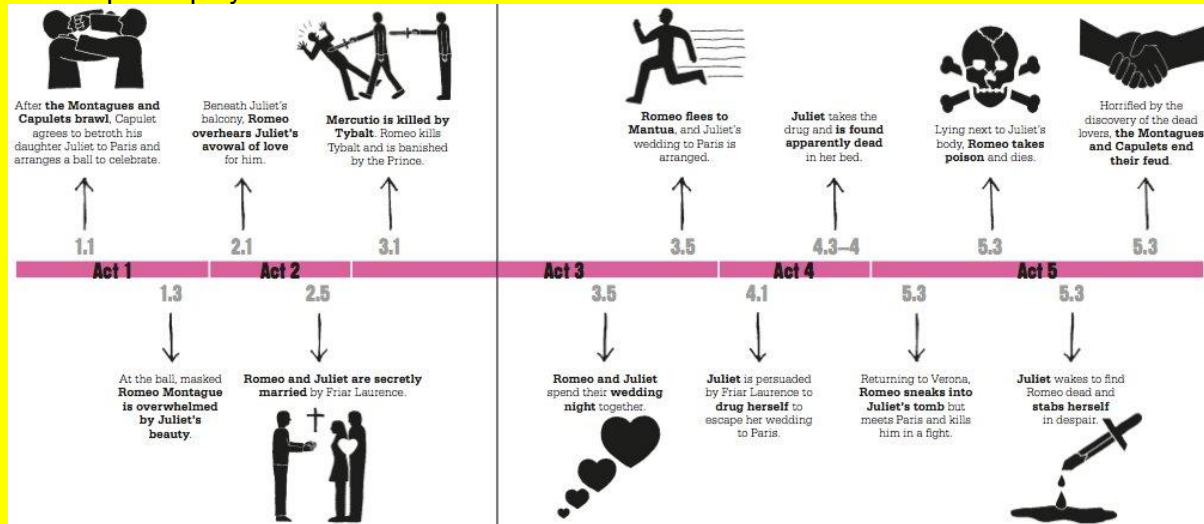
You can use timelines to find the difference in time between two events.

How long after Winston Churchill became Prime Minister did the Allies invade Italy?

The Allies invaded Italy in 1943. Winston Churchill became Prime Minister in 1940.

$1943 - 1940 = 3$ years

Timelines are also frequently used in English. This timeline shows how the plot progresses in the Shakespeare play Romeo and Juliet.



The 12 hour and 24 hour clock

Analogue clocks	Digital clocks
 <p>Analogue clocks show the time on a 12 hour clock face. The short hand indicates the hour and the long hand the number of minutes.</p> <p>For the long hand, each number represents 5 minutes.</p>	 <p>Digital clocks normally display the time using the 24 hour clock, although they can be set to display the time in the 12 hour clock as well.</p>
 <p>Nine thirty (a.m./p.m.) Half past nine</p>	 <p>Nine forty-five (a.m./p.m.) Quarter to ten</p>
 <p>Two fifteen (a.m./p.m.) Quarter past two</p>	 <p>Two fifteen (a.m./p.m.) Quarter past two</p>

Midnight

Midday

	12 hour clock	24 hour clock
Midnight	12 : 00 a.m.	00 : 00
	1 : 00 a.m.	01 : 00
	2 : 00 a.m.	02 : 00
	3 : 00 a.m.	03 : 00
	4 : 00 a.m.	04 : 00
	5 : 00 a.m.	05 : 00
	6 : 00 a.m.	06 : 00
	7 : 00 a.m.	07 : 00
	8 : 00 a.m.	08 : 00
	9 : 00 a.m.	09 : 00
	10 : 00 a.m.	10 : 00
	11 : 00 a.m.	11 : 00
Midday	12 : 00 p.m.	12 : 00
	1 : 00 p.m.	13 : 00
	2 : 00 p.m.	14 : 00
	3 : 00 p.m.	15 : 00
	4 : 00 p.m.	16 : 00
	5 : 00 p.m.	17 : 00
	6 : 00 p.m.	18 : 00
	7 : 00 p.m.	19 : 00
	8 : 00 p.m.	20 : 00
	9 : 00 p.m.	21 : 00
	10 : 00 p.m.	22 : 00
	11 : 00 p.m.	23 : 00

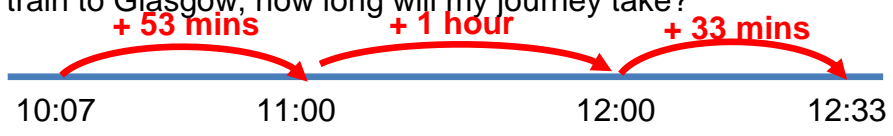
Timetables

Belfast - Stranraer - Glasgow

Mondays to Saturdays

	SX	S0					SX	S0				SX	
Belfast Port	d					0730r		1145g	1145z			1700g	1920r
Stranraer Harbour	a					1020r		1345g	1345z			1920g	2210r
Stranraer	d		070	1007		1240		1443	1443			1940	2112 2312
Barrhill	d		0743	1042		1319		1517	1517			2019	2146 2347
Girvan	d	0620	0620	0801	1101	1206	1337	1440	1536	1536	1733	1933	2037 2206 0006
Maybole	d	0636	0636	0825	1117	1222	1353	1456	1552	1552	1756	1956	2053 2223 0022
Ayr	a	0648	0648	0835	1129	1234	1405t	1508v	1604	1604	1808b	2008f	2105 2235 0034
Prestwick Town	a	0655	0655	0841	1148c	1241	1423	1523	1611	1611	1823	2022	2111 2241
Prestwick Int. Airport	a	0657	0657	0843	1150c	1243	1425	1525	1613	1613	1825	2024	2113 2308c
Troon	a	0702	0702	0848	1154c	1248	1430	1530	1618	1618	1830	2029	2118 2246
Kilmarnock	a	0716	0716	0904		1304	1453	1546	1634	1634	1846	2045	2137
Kilmarnock	d	0722	0723	0927				1557					2200
Barrhead	a	0747	0748	0952		1352	1552	1652	1722	1722	1922	2122	2220
Kilwinning	a	0719c	0736c	0904c	1149	1304c	1436c	1536c	1636c	1636c	1836c	2036c	2136c 2255
Paisley Gilmour St	a	0747c	0804c	0924c	1215	1323c	1455c	1557c	1657c	1657c	1857c	2055c	2157c 2314
Glasgow Central	a	0800	0809	1005g	1233	1335c	1508c	1633y	1709c	1709c	1909e	2107c	2234h 2325

Timetables allow us to plan journeys. If I arrive at Stranraer station at 9 am and catch the next train to Glasgow, how long will my journey take?



$$\begin{aligned}
 &53 \text{ mins} + 1 \text{ hour} + 33 \text{ mins} \\
 &= 1 \text{ hour } 86 \text{ mins} \\
 &= \underline{\underline{2 \text{ hours } 26 \text{ mins}}}
 \end{aligned}$$

2.16 Compass directions and bearings

Compass directions

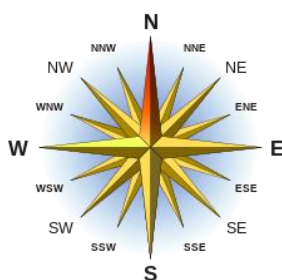
The main compass directions are North, East, South and West.

Compasses use magnets to show which direction is North.



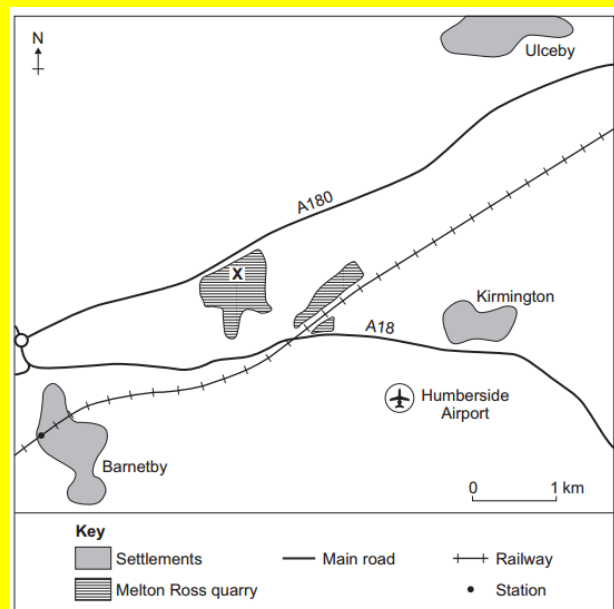
They are used to find directions when you can't use roads etc.

On maps North is usually straight up.



N : North
 NNE: North North East
 NE: North East
 ENE: East North East
 E: East
 SE: South East
 S: South
 SW: South West
 W: West
 NW: North West

Geography



Melton Ross quarry is 2 km **West** of Kirmington.

Barnetby station is 3 km **South West** of location X at the quarry.

Bearings

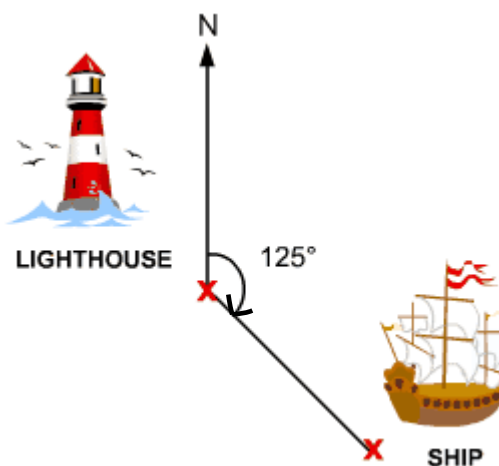
A bearing is the angle that must be turned through from North to be facing in a desired direction.

Aircraft and ships use bearings to make sure they are facing the correct direction, and therefore avoid collisions.

Bearings are always written using 3 digits.

For example, if the angle you needed to turn through from North was 5° , the bearing would be 005° .

If the angle you needed to turn through from North was 62° , the bearing would be 062° .



The bearing of the ship from the lighthouse is 125° .





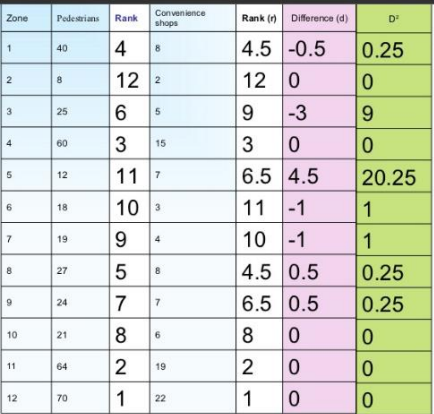
The bearing of the lighthouse from the ship is 305° .

2.17 Algebra

Algebra uses letters to represent unknown quantities or variables. Most of the algebra that students learn in school is only encountered in Mathematics lessons, but there are some elements of algebra that are used across the curriculum. These are explored here.

2.17.1 Substitution into formulae

Key points	Formula (equations) used in Physics																														
<p>Formulae are used across the curriculum. The most common examples outside of Mathematics are in science, DT and business.</p> <p>A formula basically tells you how to work out the value of something based on other things you are given. Substitution into a formula means replacing the letters in a formula with their numerical values.</p> <p>Example 1</p> <p>The formula for the area of a rectangle is:</p> $A = l \times w$ <p>Where</p> <p>A = The area of the rectangle l = The length of the rectangle w = The width of the rectangle</p> <p>If you are told that the length of a rectangle is 12 cm and that its width is 5 cm, you would calculate its area by substituting these values into the formula.</p> $A = l \times w = 12 \times 5 = 60 \text{ cm}^2$ <p>The formula can also be written as:</p> $A = lw$ <p>If two letters are written next to each other it means that their values should be multiplied together e.g. IR means $I \times R$. A horizontal line between two quantities indicates that they should be divided e.g. $\frac{Q}{t}$ means $Q \div t$</p>	<p>Some of the formulae used in Physics are listed below:</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 5px;">$a = \frac{F}{m}$ or $F = m \times a$</td> <td style="padding: 5px;">F : resultant force m : mass a : acceleration</td> </tr> <tr> <td style="padding: 5px;">$a = \frac{v - u}{t}$</td> <td style="padding: 5px;">a : acceleration v : final velocity u : initial velocity t : time taken</td> </tr> <tr> <td style="padding: 5px;">$W = m \times g$</td> <td style="padding: 5px;">W : weight m : mass g : gravitational field strength</td> </tr> <tr> <td style="padding: 5px;">$F = k \times e$</td> <td style="padding: 5px;">F : force k : spring constant e : extension</td> </tr> <tr> <td style="padding: 5px;">$W = F \times d$</td> <td style="padding: 5px;">W : work done F : force applied d : distance moved in the direction of the force</td> </tr> <tr> <td style="padding: 5px;">$P = \frac{E}{t}$</td> <td style="padding: 5px;">P : power E : energy transferred t : time taken</td> </tr> <tr> <td style="padding: 5px;">$E_p = m \times g \times h$</td> <td style="padding: 5px;">E_p : change in gravitational potential energy m : mass g : gravitational field strength h : change in height</td> </tr> <tr> <td style="padding: 5px;">$E_k = \frac{1}{2} \times m \times v^2$</td> <td style="padding: 5px;">E_k : kinetic energy m : mass v : speed</td> </tr> <tr> <td style="padding: 5px;">$p = m \times v$</td> <td style="padding: 5px;">p : momentum m : mass v : velocity</td> </tr> <tr> <td style="padding: 5px;">$I = \frac{Q}{t}$</td> <td style="padding: 5px;">I : current Q : charge t : time</td> </tr> <tr> <td style="padding: 5px;">$V = \frac{W}{Q}$</td> <td style="padding: 5px;">V : potential difference W : work done Q : charge</td> </tr> <tr> <td style="padding: 5px;">$V = I \times R$</td> <td style="padding: 5px;">V : potential difference I : current R : resistance</td> </tr> <tr> <td style="padding: 5px;">$P = \frac{E}{t}$</td> <td style="padding: 5px;">P : power E : energy t : time</td> </tr> <tr> <td style="padding: 5px;">$P = I \times V$</td> <td style="padding: 5px;">P : power I : current V : potential difference</td> </tr> <tr> <td style="padding: 5px;">$E = V \times Q$</td> <td style="padding: 5px;">E : energy V : potential difference Q : charge</td> </tr> </tbody> </table>	$a = \frac{F}{m}$ or $F = m \times a$	F : resultant force m : mass a : acceleration	$a = \frac{v - u}{t}$	a : acceleration v : final velocity u : initial velocity t : time taken	$W = m \times g$	W : weight m : mass g : gravitational field strength	$F = k \times e$	F : force k : spring constant e : extension	$W = F \times d$	W : work done F : force applied d : distance moved in the direction of the force	$P = \frac{E}{t}$	P : power E : energy transferred t : time taken	$E_p = m \times g \times h$	E_p : change in gravitational potential energy m : mass g : gravitational field strength h : change in height	$E_k = \frac{1}{2} \times m \times v^2$	E_k : kinetic energy m : mass v : speed	$p = m \times v$	p : momentum m : mass v : velocity	$I = \frac{Q}{t}$	I : current Q : charge t : time	$V = \frac{W}{Q}$	V : potential difference W : work done Q : charge	$V = I \times R$	V : potential difference I : current R : resistance	$P = \frac{E}{t}$	P : power E : energy t : time	$P = I \times V$	P : power I : current V : potential difference	$E = V \times Q$	E : energy V : potential difference Q : charge
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<p>Example 2 - Chemistry</p> <p>The maximum theoretical mass of product in a certain reaction is 20g, but only 15g is actually obtained. What is the percentage yield?</p> $\text{Percentage yield} = \frac{\text{Actual mass}}{\text{Theoretical mass}} \times 100$ $\text{Percentage yield} = \frac{15}{20} \times 100 = 75\%$																															

<p>Example 3 – Computer Science</p> <p>Calculate the file size in bits for a two minute sound recording that has used a sample rate of 1000 Hertz (Hz) and a sample resolution of 5 bits.</p> <p>Students are expected to recall the following formula:</p> <p>File size (bits) = rate x res x secs</p> <p>Where</p> <p>rate = sampling rate res = sampling resolution secs = number of seconds</p>  <p>Substituting in:</p> <p>File size = $1000 \times 5 \times (2 \times 60) = \mathbf{600000 \text{ bits}}$</p>	<p>Example 4 – Biology / PE</p> <p>Your body mass index (BMI) is an indication of whether you are overweight, underweight or a healthy weight.</p> <p>It can be calculated using the formula:</p> $BMI = \frac{\text{mass (in kg)}}{\text{height}^2 \text{ (in m}^2\text{)}}$ <p>What is the BMI of a 1.7 metre-tall person with a body mass of 60kg?</p> $BMI = \frac{60}{1.7^2} = 20.761245 \dots = \mathbf{20.8 (1 dp)}$ <p>Students are expected to be able to round BMIs to 1 decimal place.</p> <p>A BMI between 18.5 and 24.9 indicates an ideal weight.</p>
<p>Example 5 - Biology</p> <p>When using microscopes, lengths measured under a microscope need to be converted to actual lengths using the following formula:</p> $\text{Length of object} = \frac{\text{length of magnified object}}{\text{magnification}}$ <p>If a specimen appeared 10mm in length under a microscope with a magnification of 1,000 times, what would the actual length be?</p> <p>Length of object = $10 \div 1000 = \mathbf{0.01 \text{ mm}}$</p> 	<p>Example 6 – Business studies</p> <p>Profitability ratios are a way of measuring how much profit a business makes.</p>  <p><i>Gross profit percentage ratio</i></p> $= \frac{\text{Gross profit}}{\text{Net sales}} \times 100$ <p>A business has a gross profit of £200000 and net sales of £800000, what is its gross profit percentage ratio?</p> $\text{Gross profit percentage ratio} = \frac{200000}{800000} \times 100 = \mathbf{25\%}$
<p>Example 7 – Business studies</p> <p>The average rate of return compares the profit being made with the money invested.</p> <p><i>Average rate of return</i></p> $= \frac{\text{average annual return (profit)}}{\text{Initial outlay}} \times 100$ <p>An investment of £110 generated £150 over 5 years. What was the average rate of return?</p> <p>Profit over 5 years = $\pounds 150 - \pounds 110 = \pounds 40$</p> <p>Average annual profit = $\pounds 40 \div 5 = \pounds 8$ (per year)</p> <p>Average rate of return = $\frac{8}{110} \times 100 = 7.3\% (1dp)$</p>	<p>Example 9 - Geography</p> <p>Spearman's rank is a measure of the strength of the relationship between two sets of data.</p>  $r_s = 1 - \frac{6 \sum d^2}{n^3 - n}$ <p>n is the number of sites/zones. d is the difference in rank between the two sets of data.</p> <p>\sum means to sum these values.</p> $r_s = 1 - \frac{6 \times 36}{12^3 - 12} = 1 - 0.1118 \dots = \mathbf{0.888}$

2.17.2 Rearranging formulae

Key points

Rearranging a formula involves changing its subject. The subject of a formula is whatever is stated on its own equal to something else.

For example if we know the formula for calculating speed in terms of distance and time, we should be able to rearrange it so we have a formula for calculating distance in terms of speed and time. We should also be able to rearrange it so we have a formula for calculating the time in terms of speed and distance.

There are certain formulae that students are expected to be able rearrange confidently. Outside of Mathematics the main formula that students are expected to rearrange across the curriculum are:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

However students can be expected to rearrange a number of formulae in both Mathematics and Science.

Some teachers like to share “**formula triangles**” with students to help them rearrange equations. Since these triangles only work if the variables in a formula have a specific type of relationship, and often result in students making careless errors, we tend to avoid teaching them in Mathematics.

You rearrange a formula using the balance method. As long as you do the same operation to both sides of the equation it will stay balanced.

e.g. $a = c \times b$

To make b the subject, divide both sides by c:

$$\frac{a}{c} = \frac{c \times b}{c} = b$$

$$b = \frac{a}{c}$$

Example 1 - Physics

A miner has a mass of 90 kg. The change in gravitational potential energy when he moves 15 m down a slide is calculated to be 13500 Joules.

Calculate the maximum possible speed that the miner could reach at the bottom of the slide. (3 marks)



$$E_k = \frac{1}{2} \times m \times v^2$$

Substituting into the equation will get a student 2 of the 3 marks so do this first.

$$13500 = \frac{1}{2} \times 90 \times v^2$$

$$13500 = 45 \times v^2$$

Finally rearrange the equation:

$$v^2 = \frac{13500}{45}$$

$$v = \sqrt{\frac{13500}{45}} = 17.3 \text{ m/s}$$

Example 2 - Mathematics

A train travels 20 kilometres at an average speed of 85 kilometres per hour. How long does this section of the journey take?

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Multiply both sides of the equation by “time”

$$\text{speed} \times \text{time} = \text{distance}$$

Divide both sides of the equation by “speed”:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Now substitute into the formula:

$$\text{time} = \frac{20}{85} = 0.235 \text{ hours}$$

Multiply by 60 to get the time in minutes:

$$0.235 \times 60 = 14 \text{ minutes (to the nearest min)}$$

2.17.3 A note on chemical formulae in science

In science chemical formulae for different substances are given using a series of letters and numbers. The letters represent the chemical elements that make up the substance, and the numbers indicate the quantity of one element compared to another within the substance.

A number after a letter belongs ONLY to the letter immediately before it.

For example:



CO₂

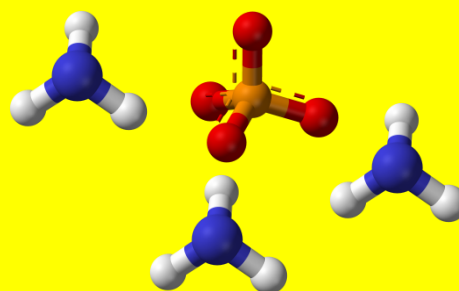
is the chemical formula for Carbon dioxide. The "2" belongs only to the "O", i.e. the Oxygen. This indicates that in a molecule of Carbon di-oxide there is one Carbon atom, for every two Oxygen atoms.

If there are numbers after a bracket, everything inside the bracket is multiplied by that number.

For example:

(NH₄)₃PO₄

Number of "N" (Nitrogen) atoms = 1 x 3 = 3
Number of "H" (Hydrogen) atoms = 4 x 3 = 12
Number of "P" (Phosphorus) atoms = 1
Number of "O" (Oxygen) atoms = 4

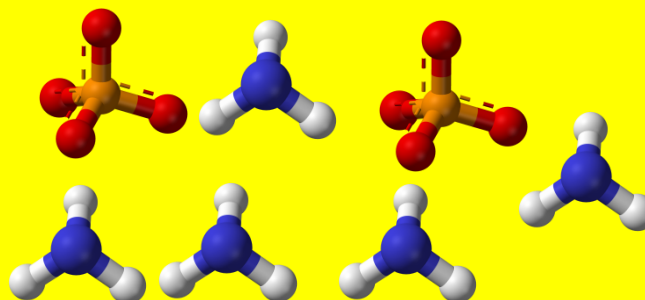


If there is a number at the START of a formula (as there often is when equations are balanced), everything after the number is multiplied by this.

For example:

2(NH₄)₃PO₄

Number of "N" (Nitrogen) atoms = 2 x 1 x 3 = 6
Number of "H" (Hydrogen) atoms = 2 x 4 x 3 = 24
Number of "P" (Phosphorus) atoms = 2 x 1 = 2
Number of "O" (Oxygen) atoms = 2 x 4 = 8



Understanding how formulae are written is crucial in being able to balance equations and carry out reacting mass calculations.

3 Handling Data

There are two main types of data:

1) Discontinuous / discrete data

Discrete data can only take particular, defined values – for example shoe size

2) Continuous data

Continuous data can take any value over a continuous range – for example height

3.1 Collecting and recording data

Methods of recording data																																															
<p>Lists</p> <p>1, 2, 1, 1, 2, 3, 2, 1, 2, 1, 1, 2, 4, 2, 1, 5, 2, 3, 1, 1, 4, 10, 3, 2, 5, 1</p> <p>Lists of data can be hard to interpret. It is therefore useful to record data in tables.</p> <p>“Frequency” tells us how many there are of something.</p>	<p>Data collection sheets (Frequency tables / Tally charts)</p> <table border="1"> <thead> <tr> <th>Number of pets</th> <th>Tally</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td>1</td> <td> </td> <td>10</td> </tr> <tr> <td>2</td> <td> III</td> <td>8</td> </tr> <tr> <td>3</td> <td> </td> <td>3</td> </tr> <tr> <td>4</td> <td> </td> <td>2</td> </tr> <tr> <td>5</td> <td> </td> <td>2</td> </tr> <tr> <td>6</td> <td></td> <td>0</td> </tr> <tr> <td>7</td> <td></td> <td>0</td> </tr> <tr> <td>8</td> <td></td> <td>0</td> </tr> <tr> <td>9</td> <td></td> <td>0</td> </tr> <tr> <td>10</td> <td> </td> <td>1</td> </tr> </tbody> </table> <p>Tallies are used to record data as you go along. Once all the data has been collected, the frequency (total) can be written next to the tally. Tallies are written in groups of 5 allowing you to quickly calculate the frequency.</p>	Number of pets	Tally	Frequency	1		10	2	III	8	3		3	4		2	5		2	6		0	7		0	8		0	9		0	10		1	<p>Grouped frequency tables</p> <table border="1"> <thead> <tr> <th>Time taken (<i>m</i> minutes)</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td>$0 < m \leq 10$</td> <td>3</td> </tr> <tr> <td>$10 < m \leq 20$</td> <td>8</td> </tr> <tr> <td>$20 < m \leq 30$</td> <td>11</td> </tr> <tr> <td>$30 < m \leq 40$</td> <td>9</td> </tr> <tr> <td>$40 < m \leq 50$</td> <td>9</td> </tr> </tbody> </table> <p>If there are lots of different values that the data can take, it is useful to group the possible values together to help summarise the results.</p>	Time taken (<i>m</i> minutes)	Frequency	$0 < m \leq 10$	3	$10 < m \leq 20$	8	$20 < m \leq 30$	11	$30 < m \leq 40$	9	$40 < m \leq 50$	9
Number of pets	Tally	Frequency																																													
1		10																																													
2	III	8																																													
3		3																																													
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$20 < m \leq 30$	11																																														
$30 < m \leq 40$	9																																														
$40 < m \leq 50$	9																																														

3.1.1 Sampling and sample size

It is not always practical to collect data from every member of the population you are investigating. For example if you were collecting data about whether men smoked in Hyde, it would not be practical to ask every single male who lived in Hyde. In situations like this it is easier to take a sample of the population and to ask them. In general, the larger your sample size the more accurate a picture your data will give you of the whole population. There is therefore a balance between choosing a sample size which is accurate enough, while still being practical to use.

3.1.2 Control groups and reliability

If people take part in a clinical trial, their expectations can influence the results. Volunteers for clinical trials therefore tend to be put into two groups at random. Checks are done to make sure both groups have a similar gender balance and age range.

The two main types of clinical trial are summarised below. In both trials one group of volunteers, called the test group, receives the new drug. Another, the **control group**, receives the existing drug for that illness or a fake drug that has no effect on the body, called a placebo. The researchers look for differences between the experimental group and the control group.

Blind trials	Double-blind trials
In a blind trial, volunteers do not know which group they are in but the researchers do. The problem is the researchers may give away clues to the volunteers without realising it. This is called observer bias. It can make the results unreliable.	In a double-blind trial, the volunteers do not know which group they are in, and neither do the researchers, until the end of the trial. This removes the chance of bias and makes the results more reliable. But double-blind trials are more complex to set up.

3.2 Displaying data

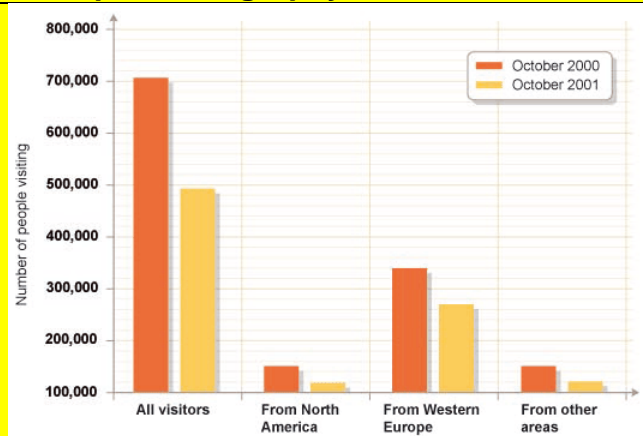
3.2.1 Bar charts

Bar charts are one of the most common ways of representing data across the curriculum. They are particularly useful for data with **discontinuous** variation – i.e. when the data can only take specific values. An example of discontinuous data is blood types.

Key points

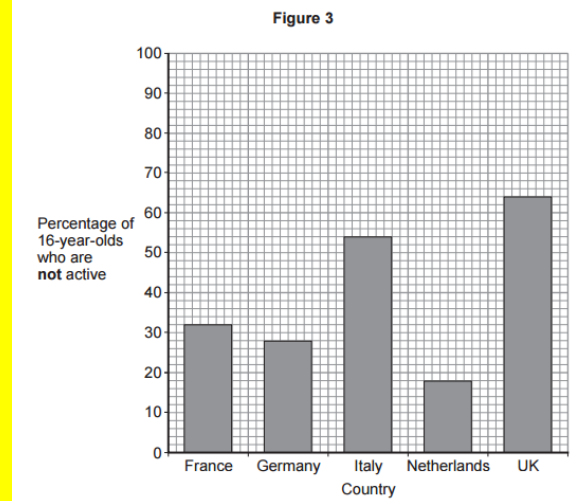
- The height of each bar represents the frequency. The vertical axis is often labelled “frequency”.
- All bars must be the same width and there must be equal sized gaps between the bars.
- Each bar should be clearly labelled.
- The scale on the vertical axis must be evenly spaced.

Example – Geography

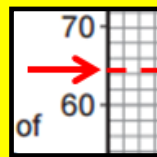


This comparative bar chart allows you to see the decline in tourism between two years.

Example - Biology



What percentage of 16-year-olds in the UK are not active?

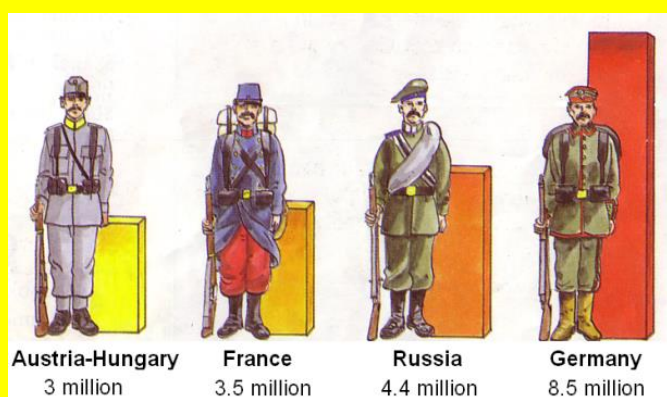


Looking at the scale on the vertical axis, we can see that every 5 squares represents 10%.

Therefore each square represents 2%.

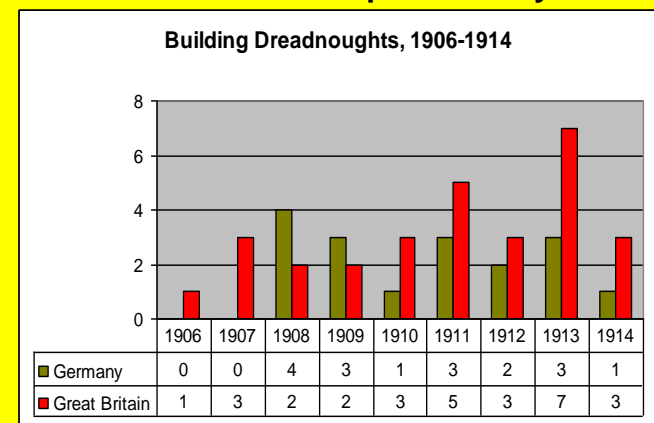
The percentage of 16-year olds in the UK who are not active is therefore **64%**.

Bar charts without scales - History



Some subjects use bars to allow a visual comparison of data, but don't use a scale. They show the values below the bar instead.

Duel bar charts – Example - History



Duel bar charts allow you to compare two sets of data. This duel bar chart compares the number of Dreadnought submarines Germany and Great Britain built in each year leading up to the start of World War I.

3.2.2 Histograms

Key points

In histograms the frequency is represented by the area of a bar, rather than its height.

It is very easy to confuse histograms with bar charts.

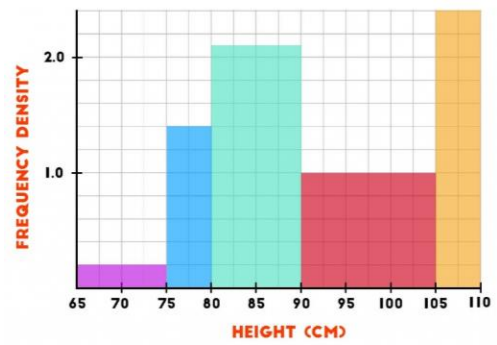
Unlike bar charts, histograms do not have gaps between their bars. This is because they are drawn for grouped **continuous** data – meaning that the data can take any value in a given range.

Since it is the area of the bar that gives the frequency, in a histogram the widths of the bars do not have to be the same.

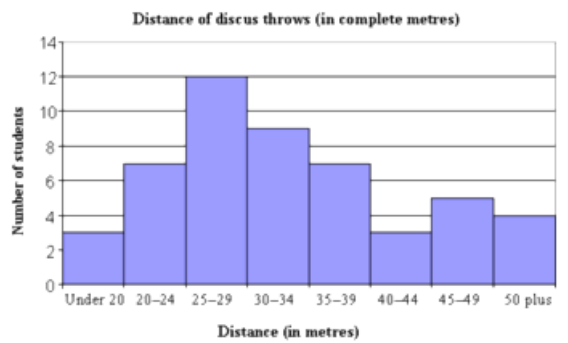
The vertical axis should be labelled frequency density.

For curriculum areas outside of Mathematics, histograms normally have equal width bars. However the vertical axis in these histograms is often incorrectly labelled as frequency, rather than frequency density.

Example - Mathematics



Example – PE

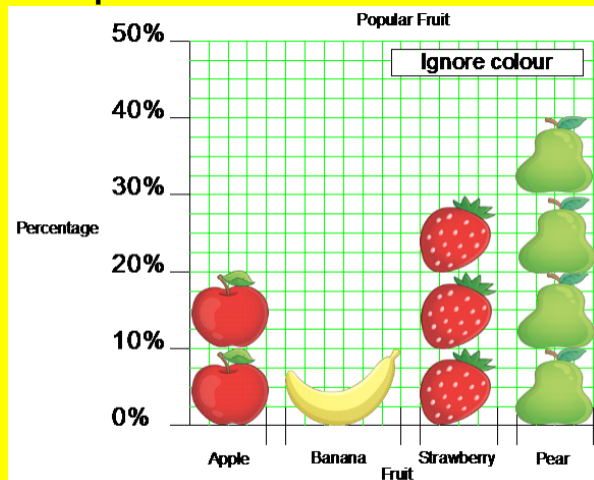


3.2.3 Pictograms / Pictographs

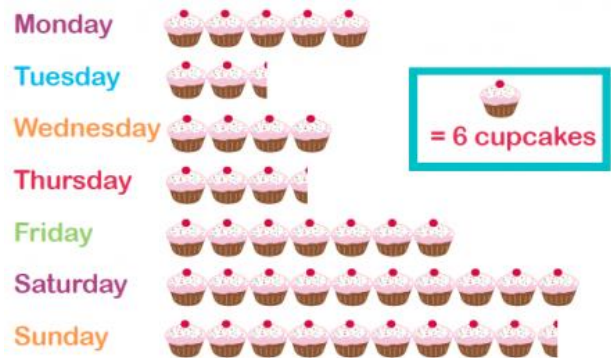
Key points

- A pictogram uses symbols to represent frequency.
- We include a key to show the value of each symbol.
- In some subject areas Pictograms are used in a similar way to a bar chart (see the DT example below)

Example - DT



Example



The key tells us that each cupcake symbol represents 6 cupcakes.

That means that each half cupcake symbol represents 3 cupcakes.

Next to Tuesday there are 2 and a half cupcakes.

That means on **Tuesday** $6 + 6 + 3 = \underline{15}$ cupcakes were eaten.

3.2.4 Pie charts

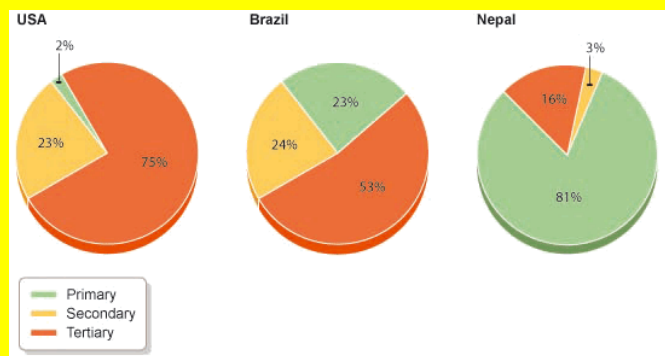
Key Points

The complete circle represents the total frequency,
 A full turn is 360° , so the angle for each sector is calculated by first of all working out what fraction of the total each group is, and then finding that fraction of 360° .

Example - Geography

While students are rarely asked to draw Pie Charts in curriculum areas other than Mathematics, they are often asked to interpret them.

These pie charts show the percentage of different types of employment in three different countries.



From the charts it is clear that the USA had the greatest percentage of tertiary employment while Nepal had the least.

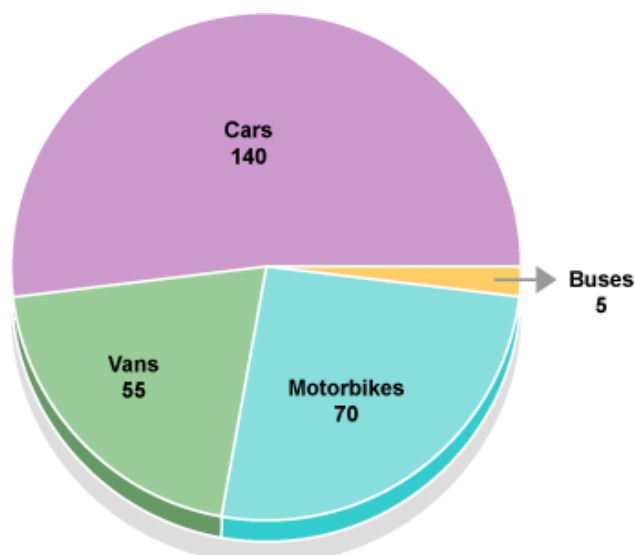
What the pie charts don't tell you is the number of people each sector represents, just the relative amounts.

We can see that the USA had a greater percentage of tertiary employment than Nepal but without knowing the actual populations of the country we can't tell whether this represents more people or not.

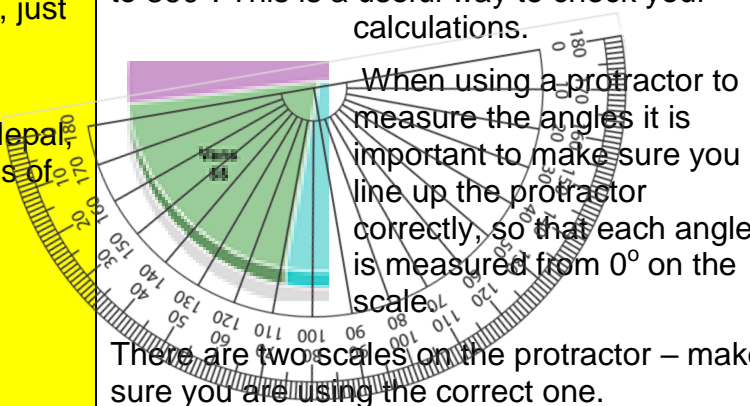
Example

A traffic survey was carried out. The results were as follows:

Type of vehicle	Number of vehicles	Angle
Cars	140	$\frac{140}{270} \times 360 = 187^\circ$
Motorbikes	70	$\frac{70}{270} \times 360 = 93^\circ$
Vans	55	$\frac{55}{270} \times 360 = 73^\circ$
Buses	5	$\frac{5}{270} \times 360 = 7^\circ$
	270	360°



The angles should, if calculated correctly, sum to 360° . This is a useful way to check your calculations.



There are two scales on the protractor – make sure you are using the correct one.

Make sure to label each sector of the pie chart clearly or/and to use a key.

3.2.5 Line graphs

Key points

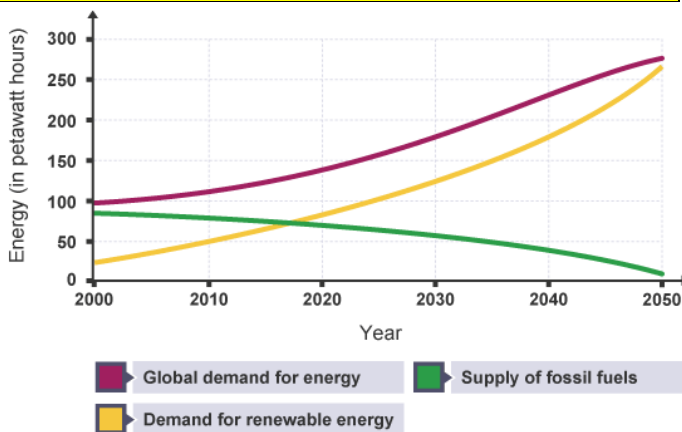
Line graphs are used across the curriculum to show how one variable changes as another one is increased.

They are particularly useful in showing how things change over time.

The two variables are represented along the horizontal and vertical axis. Data is plotted in points and the points are then joined with straight lines or a smooth curve as appropriate.

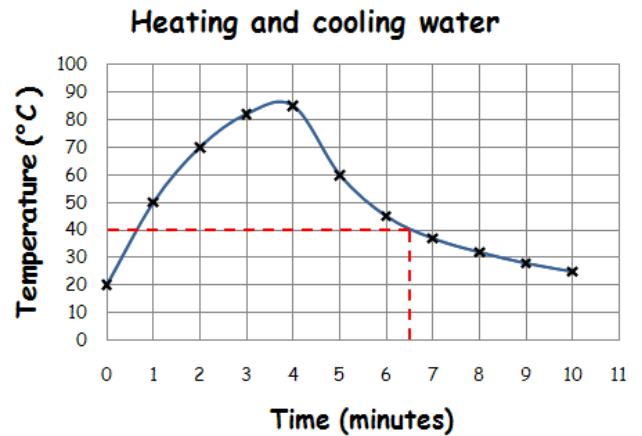
Line graphs can also be used to show predictions for how things will change in the future.

This example from Geography shows how the demand for energy / renewable energy is predicted to change in the next 40 years along with the supply of fossil fuels.



Example - science

In an experiment, the temperature of water was measured every minute as it was heated and then left to cool. The data was recorded on a graph as shown and a line graph drawn to show the pattern over time.



The line enables us to estimate the temperature of the water at times other than those plotted

e.g. at 6½ minutes the temperature was approximately 40 °C.

The line can also be extended to make predictions about how the temperature of the water will change in the future.

Outliers (anomalies / rogue values)

An **outlier** is a data point that lies far away from the general trend of the data.

It is possible that it has come from an inaccurate measurement of the data or from a different process that generated the rest of the data points.

Outliers are usually ignored when identifying trends in the data, drawing lines on a line graph or drawing a line/curve of best fit on a scatter graph.



See Appendix 3 for how line graphs can be used to analyse English texts.

Choosing a suitable scale

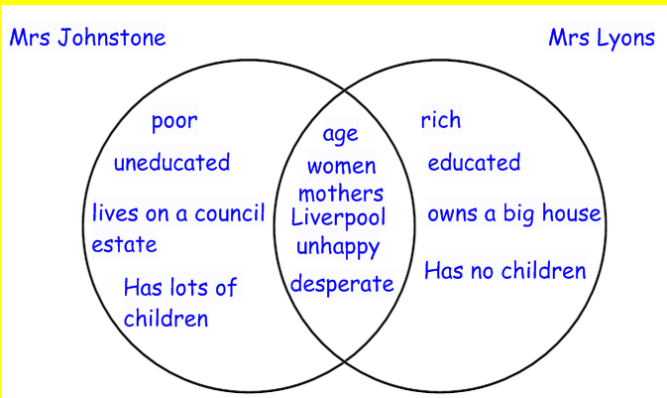
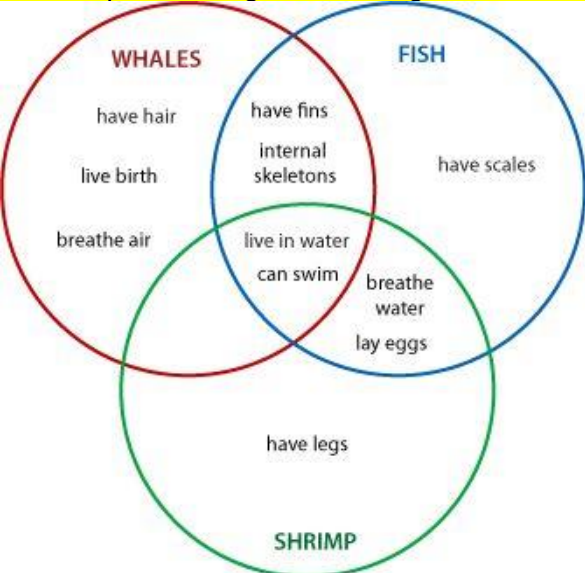
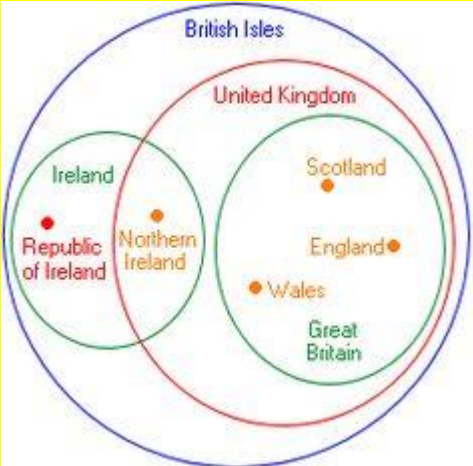
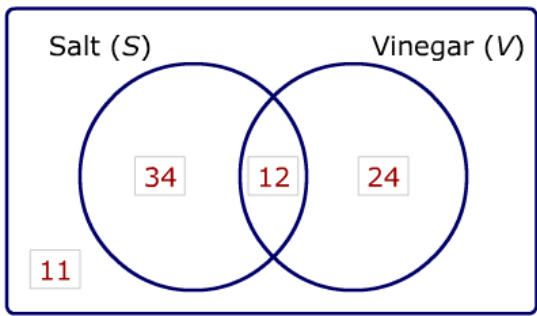
The scale on a line graph refers to the values on the vertical or horizontal axis and how they are spaced.

Often in Mathematics assessments students will be given scaled axes to use or be allowed to use a scale of their choice as long as it is correct.

In Science students are expected to be able to choose a **suitable** scale for their graphs. The graph should cover at least 50% of the paper, should start at 0 and should increase in equal increments, i.e. each box should be worth the same value.

3.2.6. Venn diagrams

Venn diagrams can be used to sort both numerical and non-numerical data. Data is grouped in sets. Where the sets overlap is called an intersection. Any data in the intersection is common to both sets.

Example - English	Example - Science
<p>This Venn diagram has been used to compare two of the principal female characters in the play Blood Brothers:</p> 	<p>Here the characteristics of whales, fish and shrimp are compared using a Venn diagram:</p> 
<p>This Venn diagram looks at the composition of the British Isles.</p> 	<p>This Venn diagram has recorded how people like their chips in a café:</p>  <p>12 people like both salt and vinegar. 46 people in total like salt (34 + 12). 11 people like neither salt nor vinegar.</p>

3.2.7 Scatter graphs (Sometimes referred to as line graphs in science)

Key Points

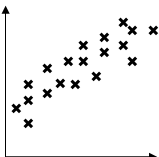
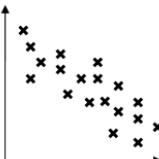
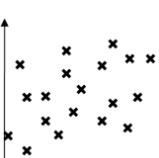
We plot points on the scatter diagram in the same way as for the line graph.

One variable is plotted along the horizontal axis, the other along the vertical axis.

We do not join the points but look for a correlation (relationship) between the two variables.

If there is a correlation, we can draw a line of best fit on the diagram and use it to estimate the value of one variable given the other.

There should be approximately the same number of points above and below the line.

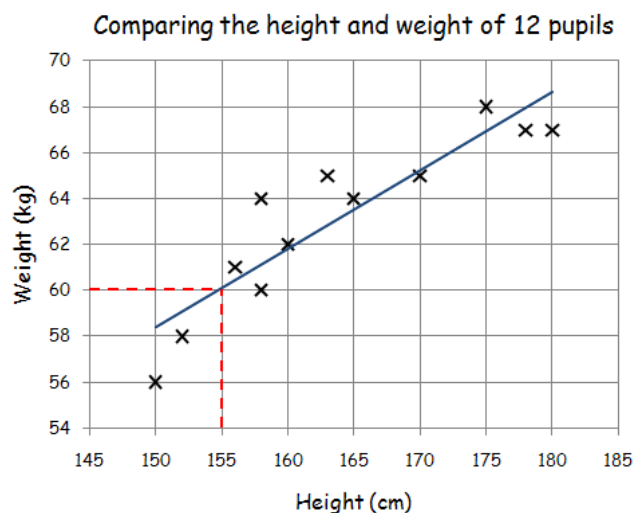
Type of correlation	Typical graph	Trend observed
Positive correlation		As one variable increases, the other variable increases.
Negative correlation		As one variable increases, the other variable decreases.
No correlation		There is no obvious relationship between the variables.

The data on scatter graphs is referred to as **bivariate data**, as each data point involves two variables.

Example

The heights and weights of 12 pupils were recorded and plotted on a scatter graph as shown below. Each point represents the height and weight of a different pupil.

A line of best fit was drawn to show the general trend of the data.



What is the relationship between the height and the weight of a child?

“As a child’s height increases so does its weight. There is a positive correlation between height and weight.”

Estimate the weight of a child who is 155 cm tall.

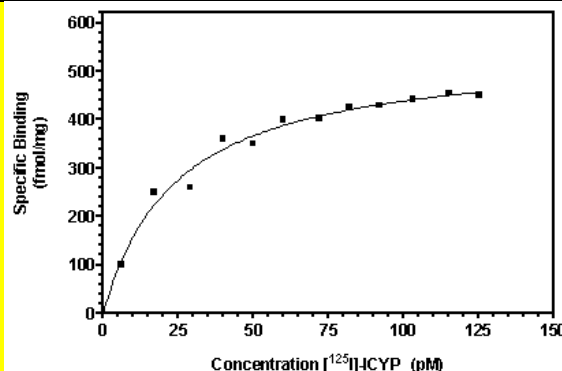
Using the line of best fit as shown, a child with a height of 155 cm would have an estimated weight of **60 kg**.

Lines of best fit – Science

In science it may be more appropriate to draw a line of best fit that is a curve as opposed to a straight line.

Which one you use depends on the context of the data collected.

For the data shown in this experiment it is clear that as the concentration is increased the specific binding is tending towards a maximum value, a curved line of best fit is therefore more appropriate.



3.2.8 Conversion graphs

Key points

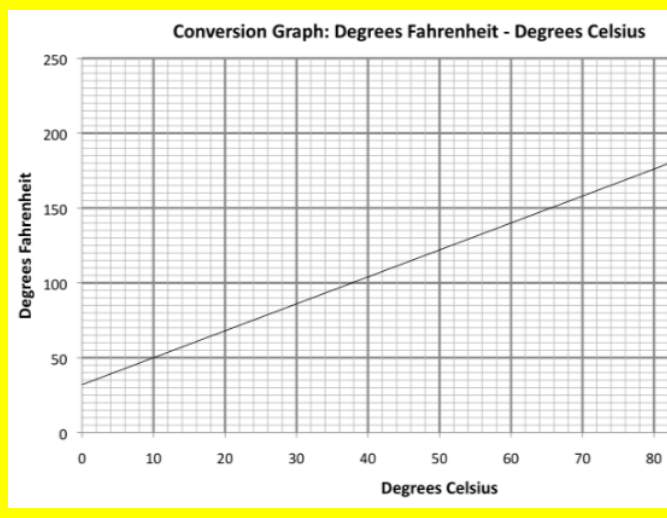
We use a conversion graph for two variables which have a linear relationship.

We draw it in the same way as a line graph.

The points are always connected with a straight line.

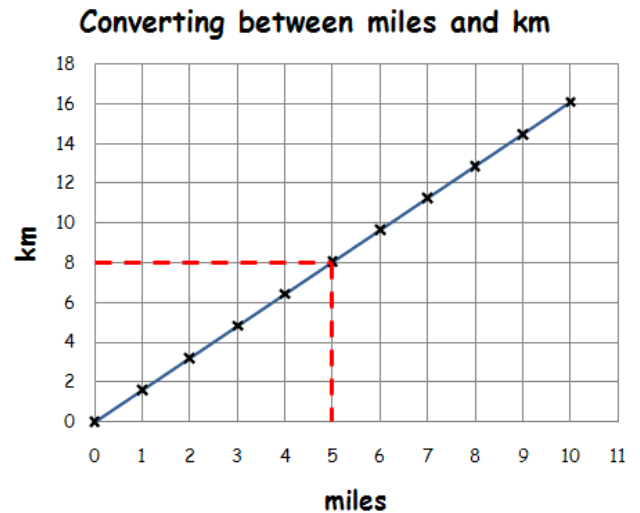
Conversion graphs can be used to establish the relationship between two variables.

This conversion graph shows the relationship between temperatures in degrees Celsius and temperatures in degrees Fahrenheit.



Example

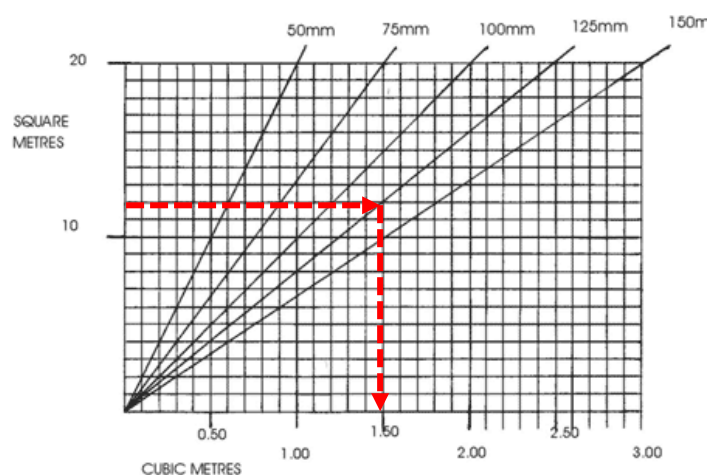
The relationship between miles and kilometres can be shown on a conversion graph.



The graph allows us to convert a distance in miles to kilometres and vice versa.

For example it can be seen that 5 miles is approximately 8 kilometres.

Example - Industry



Conversion graphs are often used in industry.

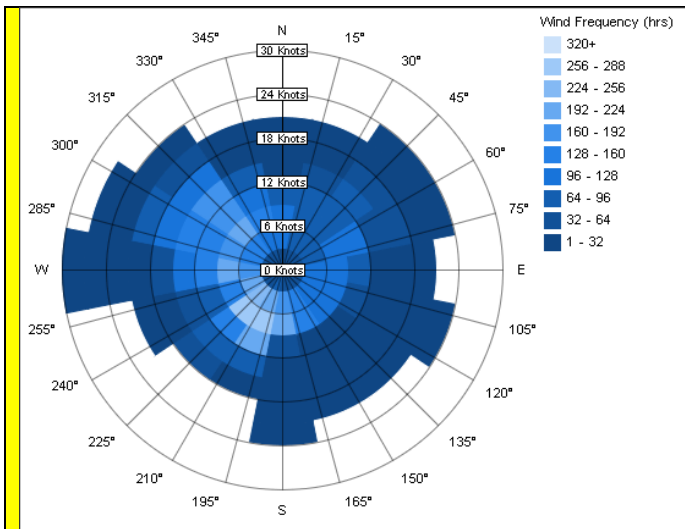
This conversion graph allows you to work out the volume of material you will need to cover a given area to a specified depth.

For example if you wanted to cover your lawn with top soil to a depth of 125 mm:

First work out the area of your garden, for example it could be 12 square metres.

Then go across from 12 on the left hand side until you meet the line representing a depth of 125 mm and go down to the horizontal axis to find the volume of soil needed, in this case 1.5 cubic metres.

3.2.9 Wind rose diagrams - Geography

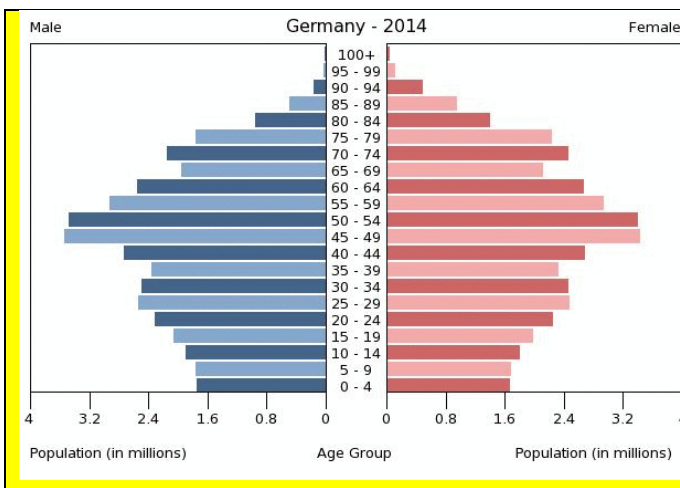


Wind rose diagrams are used in Geography to give a visual representation of wind patterns at a site.

They can be created for a specific year or season.

Wind rose diagrams help with planning the development of sites in terms of building design.

3.2.10 Population pyramids - Geography



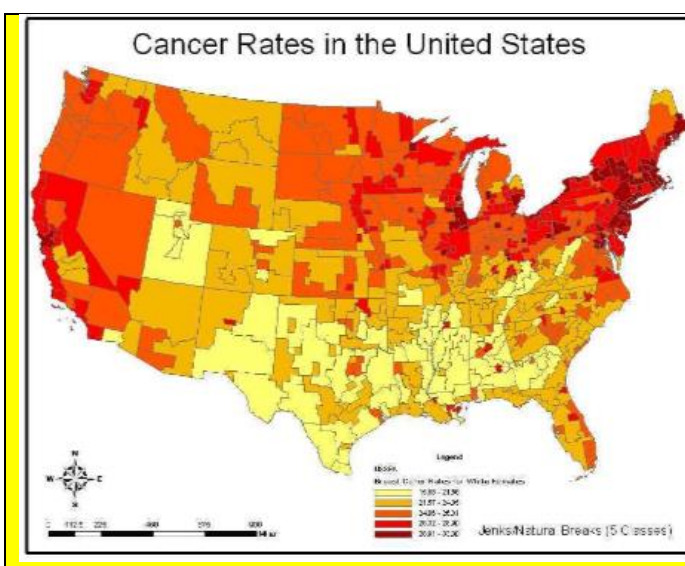
A population pyramid shows the distribution of ages in a given population for both males and females.

It can be helpful in understanding how a population is likely to change in the future.

A growing population will typically have the highest percentages of people in the younger age groups.

This pyramid for Germany shows a fairly stable population.

3.2.11 Choropleth maps - Geography



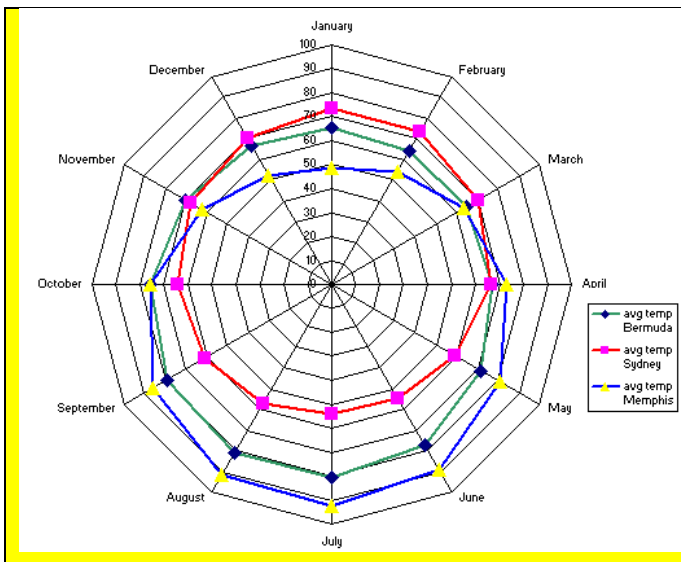
Choropleth maps are maps which use different coloured shading to show the average values of a particular quantity in different areas.

This map shows the different cancer rates across the United States.

The darkest red areas show the highest cancer rates.

The choropleth map shows us that there is a significant clustering of high cancer rates on the North East coast of the United States.

3.2.12 Radial graphs - Geography

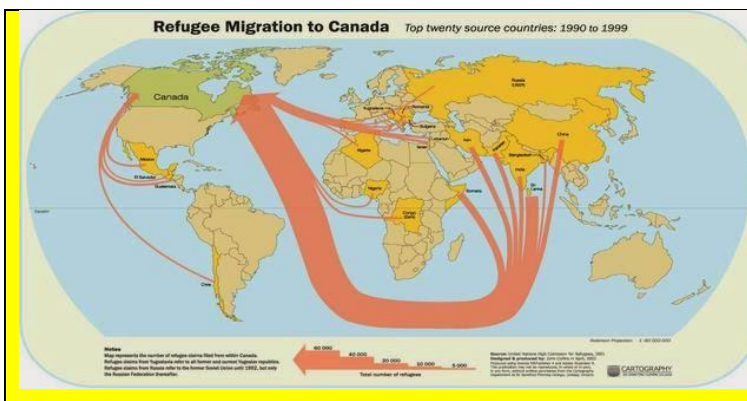


Radial graphs show information in a circular pattern. Each line going out from the centre represents a different aspect – in the example shown each line represents a different month of the year.

Information for each group is then plotted for each aspect around the circle and the points joined with straight lines.

This graph shows how the average temperatures vary over a year for 3 different cities.

3.2.13 Flow line maps - Geography

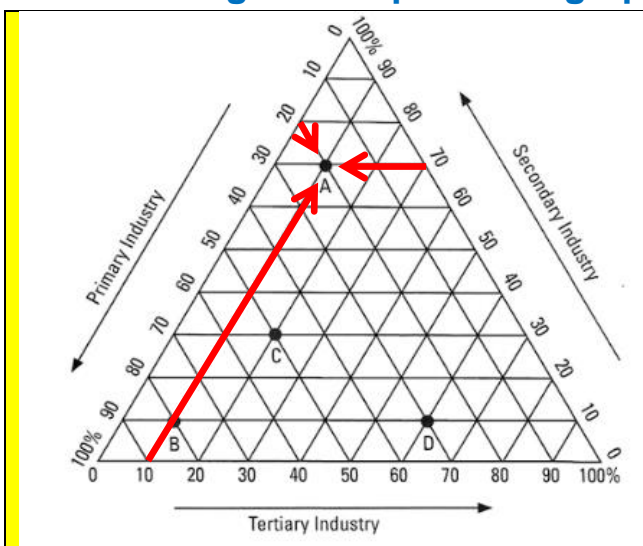


Flow line maps / diagrams show the movement of objects from one place to another.

Movement is shown with arrows, the thicker the arrow the greater the movement.

This flow line map shows migration to Canada.

3.2.14 Triangular Graphs - Geography



Triangular graphs are used in Geography to show the composition of different things. The information is read in 3 directions.

Here the right scale is read horizontally, the left scale is read on a downwards slope and the lower scale is read on an upwards slope.

The triangular graph shown is used to work out the percentage of different types of industry.

In country "A", 20% of the working population are based in Primary Industry, 70% are based in Secondary Industry and 10% are based in Tertiary Industry.

In Geography, students often struggle with interpreting the data they are given e.g. with development indicators such as understanding that a higher life expectancy and a lower birth rate are both indicators of a more developed country.

3.3 Averages

Averages give us typical values for a set of data. For example the average temperature in a city in January would be the typical temperature for that city in January. There are different types of averages.

The mean

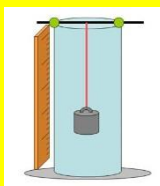
This is the average that students are most likely to be asked to calculate in different areas of the curriculum.

You find the mean by adding together all the values in a set of data and then dividing the total by the number of values you had.

Example – Science

The table below shows the length that equal-sized samples of one type of rubber can be stretched to before they break.

Sample number	1	2	3	4	5
Length (mm)	27	24	26	25	23



The mean gives a best estimate of the length that this rubber can be stretched before it breaks.

What is the **mean** for this set of data?

$$27 + 24 + 26 + 25 + 23 = 135$$

$$135 \div 5 = \underline{\underline{27 \text{ mm}}}$$

The median

The median is the middle value in a set of ordered data. To find the median simply put the data in order and find the middle value. If there are two middle values, find the value exactly half way between them.

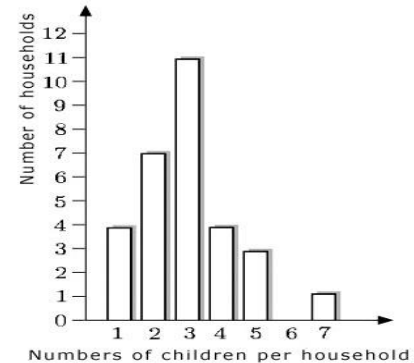
The mode

The mode is the most common value in a set of data. It is the only average that can be found for non-numerical data.

For example if you had collecting data for eye colour, the modal value would be the most common eye colour.

Bar charts can be used to find modal values – see the example to the right.

This bar chart shows how many children were in different households.



What was the **modal** number of children per household?

The highest frequency (bar) was for 3 children, so the modal number of children per household is **3**.

The range

The range gives us an indication of how spread out the data is. In Mathematics it is the difference between the largest and smallest values in a set of data.

For example if the tallest child in a class was 165 cm tall, and the shortest child was 148 cm tall, the range of heights would be given by:

$$165 - 148 = \underline{\underline{17 \text{ cm}}}$$

In science you are allowed to state the range differently.

Type of test or trial	Preclinical	Clinical phase 1	Clinical phase 2	Clinical phase 3
Tested or trialled on	Cells, tissues or animals	20-100 healthy volunteers	100-500 volunteer patients	1000-5000 volunteer patients

For this set of data, the range of volunteers needed to complete the clinical trials is given by:

$$(20 + 100 + 1000) \text{ to } (100 + 500 + 5000)$$

The **range** of volunteers needed is **1120 to 5600**

The range can, however, also be given as

$$5600 - 1120 = \underline{\underline{4480}}$$

3.3.1 Quartiles, percentiles and cumulative frequency

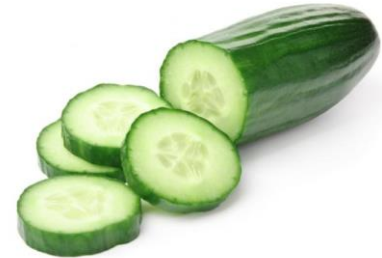
Cumulative frequency is defined as a running total of frequencies. Cumulative frequency can also be defined as the sum of all previous frequencies up to the current point.

The cumulative frequency is important when analysing data, where the value of the cumulative frequency indicates the number of elements in the data set that lie below the current value.

Example

The table shows the lengths (in cm) of 32 cucumbers.

Length (cm)	Frequency	Cumulative Frequency
21-24	3	3
25-28	7	10 (= 3 + 7)
29-32	12	22 (= 3 + 7 + 12)
33-36	6	28 (= 3 + 7 + 12 + 6)
37-40	4	32 (= 3 + 7 + 12 + 6 + 4)



Looking at the table we can see that 22 cucumbers measured less than 33 cm. Knowing the cumulative frequencies can help with quality control of products.

Quartiles, the median and percentiles

Key points	Example
<p>The lower quartile is the value below which $\frac{1}{4}$ of the data lies. This is also the 25th percentile.</p> <p>The upper quartile is the value below which $\frac{3}{4}$ of the data lies. This is also the 75th percentile.</p> <p>The median is the value below which $\frac{1}{2}$ the data lies. This is also the 50th percentile.</p> <p>In a similar way, the 10th percentile is the value below which 10% of the data lies.</p>	<p>Here is a set of numbers: 11, 4, 6, 8, 3, 10, 8, 10, 4, 12 and 31</p> <p>If we are finding the quartiles, the median or percentiles we must first put the data in order: 3, 4, 4, 6, 8, 8, 10, 10, 11, 12 and 31</p> <p>The median, or 50th percentile, is given by the middle value as $\frac{1}{2}$ the numbers will be below this.</p> <p>3, 4, 4, 6, 8, 8, 10, 10, 11, 12, 31</p> <p style="text-align: center;">↑ median</p> <p>The lower quartile, or 25th percentile, is given by the value $\frac{1}{4}$ of the way along the list, as $\frac{1}{4}$ of the values will be below this.</p> <p>The upper quartile, or 75th percentile, is given by the value $\frac{3}{4}$ of the way along the list, as $\frac{3}{4}$ of the values will be below this.</p> <p>3, 4, 4, 6, 8, 8, 10, 10, 11, 12, 31</p> <p style="display: flex; justify-content: space-around;"> ↑ lower quartile ↑ upper quartile </p>
<p>Example - science</p> <p>Percentile charts are often used to measure growth. Each line represents a different percentile. The average value for the population is the median, or the 50th percentile.</p>	<p>Anthropometric data - DT</p> <p>When designing products it is important to make sure your design is suitable for the majority of the population. Anthropometric data gives information about the size of people across a population for example their heights, weights or hand spans. Products are generally designed for the middle 90% of the population, discounting those below the 5th percentile and those above the 95th percentile, in order to help make them cost effective.</p>